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Educators who wait for utopia are obstructing progress. When we are sure of the goal, we can try for it, even with the aid of crutches.—A. J. CARLSON.

CONTENTS FOR MARCH, 1946

SCHOOL SCIENCE AND MATHEMATICS

VOL. XLVI MARCH, 1946 WHOLE NO. 401

OUR LAND AND OUR LIVING*

AN EDUCATIONAL APPROACH TO SOIL CONSERVATION

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Most people realize that the conservation of soil is a problem, the solution of which is fundamental to the ultimate welfare of human beings. There is no need to argue the necessity of anything that is now recognized and so ardently advanced. There are many signs that an awakening has come and that at least the idea of soil conservation has grown up and is of age. Many people are for it. It is generally believed to be desirable. There is a lot of talk about it, and some hopeful action is taking place. The danger is that we will only talk about soil conservation in comfortable generalities while at the same time land is becoming exhausted and people are becoming more miserable. I would like to suggest some things that should become a part of the thinking processes of the general populace, so that the present movement may be kept from slipping away in mere words. There is still too little conservation. It is either not understood or something else is considered more important.

The general aim or purpose now should be to bring about a situation where society will consciously adopt, enforce, and maintain a rationalized and, as nearly as possible, a self-perpetuating program of land use. This can be attained by changing our social philosophy and techniques through the schools as well as by other means. But the schools have a definite part.

* An address to the Conservation Group of the Central Association of Science and Mathematics Teachers, November 24, 1945.

When one scans the field of conservation in perspective, it appears that we are trying to get conservation in two ways. One way is by legislative and governmental control, imposed from above, and often accompanied by subsidy. The other way is by education and voluntary cooperation developed from within. Examples of subsidy and legislative control are the soil conservation programs where governmental technicians give service without cost, the exemption of woodland from taxes, free game and trees from state nurseries, and the crop control program. The compulsory teaching of conservation, the 4-H Club Conservation Projects, and Out-of-Door Conservation Schools are examples of education.

Whatever the approach may be, the ultimate issue in soil conservation, as in other social problems, is whether the mass-mind wants to comprehend the world in which it lives, or granted the desire, has the ability to do something about it. I go on the assumption that a sufficiently informed society, by changing its wants, can also change the factors bearing on land. We are either successful or less successful in our relationship to land in proportion to how we have learned to produce, build, and grow without destroying the base of future existence. In a democracy it must be done by education. With us, people must learn how to come to terms with nature, for without that, our democracy cannot have a permanent existence, no matter how much we may believe in it or how enthusiastically we will support it.

The many evidences of soil deterioration in this country indicate a fundamentally wrong way of living which may eventually lead to economic decline and social ruin. People need to be given an insight into the soil processes, so that they will understand how land functions. The way to stop soil deterioration is in terms of people. They come first. Everyone ought to know something about the relation of land to living. The principal difficulty in our culture is the fact that too many people have not been made aware of what is happening. They do not know the full implications of the wrong use of land, nor have they been shown a way out. We need soil conservation technicians, but most of all we need people who want technicians to show them what to do. There is a need for cultivated minds rather than trained ones.

The task facing education is to get some of the necessary conservation ideas into the thinking processes of the people. What

is required mostly is that the students in schools be made more aware of the forces that have brought land about and by which it is maintained. They need to know the causes and the reasons. It is futile to ask people to work passively for conservation without understanding the forces behind it.

Science teachers, especially biology teachers, have a unique opportunity to teach soil conservation. No other teacher is as well qualified to teach the biological concept of land so necessary in understanding the relation of living things to their environment, including soil, plants, animals, air, and water.

A cultivated field or an entire state is a biological organism, and as such is subject to the laws governing the organic. It has its critical point of effectiveness. Many factors bear on the use and the functioning of that piece of land, such as the mineral substances, organic substances, weather, erosion, and kinds of crops. A piece of land is a living entity whose capacity for production has a definite limit, as does a machine, but the kind of thinking used in machine production must not be applied in agriculture. Yet our machine age has brought that type of an agriculturist. Too often such a farmer does not recognize and guard the natural standard of the life of the soil, as he does his farm animals, not realizing that land is also an organism. The biology teacher could give his students a truer picture of soil and living things in and on it. This has not been done to any great extent. Perhaps it is because not all biologists are also ecologists, understanding some of the relationships between living things and their surroundings. Land must not be conceived as merely soil, but as an energy circuit including soil, plants, animals, water, and air. The kind of a teacher that conservation needs is one that can take a biotic view of land and transfer that view to his students.

Teaching a biological concept of land would enable the student to appreciate the forces which combine to form soil and maintain its productive capacity. The student will become conscious of the place of plants and animals in maintaining soil health. The biological view centers values on innate things rather than the artificial, such as ink marks on ledgers, rows of figures which govern the money-minded. It reveals how sensitive land is to manipulation and shows that only such management, which is biologically correct, is economically sound in the long run.

Along with teaching a biological concept of land, there are

some other ideas that should become a part of the thinking process. The following are some objectives that should be considered in conservation education.

a. *A better understanding of natural resources.* The average man has only a rudimentary notion of what resources are. I find this very prevalent among young people with whom I come into contact. Too few know that soil, iron-ore, water, oil, quarries, and mines are our real wealth, and that money is not wealth.

b. *Sensitize more people to evidences of waste.* It is human nature to waste. Everywhere I go evidence of waste may be seen. Even among the "Pennsylvania Dutch," among whom I was reared and who have the reputation for not being wasteful, there is waste. We become used to a defaced landscape and tolerate it. There are those who think we are wealthier after this war because everybody has more money. This is not so. Our county is much poorer berause of the transfer of soil nutrients, iron-ore, copper, oil, coal, etc.

c. *Give up the idea that resources are inexhaustible.* Most of them can be exhausted. Even our lakes, rivers, and subterranean waters are in danger.

d. *We need to get rid of the notion that science is a substitute for resources.* One of the greatest hindrances to soil conservation is the belief that no matter what is done, scientists and inventors will provide new substances. Science has swapped a lot of materials. It has taught us how to use resources, but often we do not realize the full implications. I have met many an enthusiast of chemurgy who saw only the money to be derived from using farm products without any appreciation of the amount and kinds of soil nutrients that left the place with them. Too few realize that we do not get anything for nothing.

e. *Teach an appreciation of land use capability.* A basic principle that should be expounded is the use of each acre according to its individual needs. Land that is suited for pasture or for woodland should not be used as cropland; and on cropland, rotations should be followed which maintain a productive capacity instead of impoverishing it.

f. *Learn what is meant by "making it pay."* Every farmer is interested in making farming pay. Rightly so. Often, however, he does not know the true meaning of what is meant by "making it pay." Does it pay to devastate the land and the environment, spending as little as possible and taking out as much as possible? Or does it pay the owner who believes it his duty to pass the

farm to the next generation in as good or better condition than when he received it? Ideas, like men, can become dictators. To be too money-minded may dictate action that may hurt land.

g. *Develop a philosophy of the right and wrong use of land.* There is no strong ethic dealing with man's relation to land. Land is property which we buy and sell at will. But can or shall we do with it as we please? When land is used so that it is detrimental to the welfare of society there should be some restraint or social pressure against that use. It should arouse our sense of right and wrong. Ethical criteria having been extended to many fields of conduct could be extended more forcibly to the use of land.

h. *Teach the skills and techniques of soil conservation.* The techniques for maintaining a productive capacity and the skills for the control of erosion should be taught better.

When thinking of the problems and the things to be taught, we must constantly keep in mind the many influences and forces *on* the farm and from *outside* the farm that often prevent man from doing the things he knows to be correct.

There are problems on the farm, such as (a) the needs of the farm family (Should it be a large farm or a small farm?); (b) the arrangement of the fields; (c) the combination of crops and crop rotations to supply livestock and soil needs; (d) the number and kind of livestock; (e) the labor needs; (f) the use of lime and fertilizer; (g) the special practices to control soil erosion; and (h) the lassitude and ignorance (Does the farmer care how he farms?).

Then there are the influences from outside the farm, such as, (a) military demands; (b) security of tenure; (c) prices, tariffs, and bounties; (d) credit facilities; (e) taxation; (f) distribution of electric power (Building of reservoirs and power dams); (g) manufacture of fertilizers; (h) transportation; (i) research and extension of results (Research and extension have raised the standard of living to unheard-of heights). Have they ever caused land disorder?

I do not know the answers to the many sociological problems. I am not wise enough to say how large a farm should be or whether farming is a way of life, or whether the purpose of farming is to produce food as Arthur Moore, editor of *Prairie Farmer*, suggests. I do know some things that are necessary if a piece of land is to maintain a productive capacity.

Producing large and excellent crops on land is an achievement,

certainly, but when it leads to the destruction of land it is, quite simply, accursed. Just what has happened and is now happening to our land in spite of our science? Isn't there a need to pay greater heed to the consequences of our science? Is it really necessary to throw the landscape so out-of-order? What is the nature of the process by which men destroy land? What is the major strategy of American agriculture?

We need to teach that a national economy of permanence cannot be built on an agriculture that operates on a policy of selling its soil as a commodity. We need to teach an insight into the soil processes—a functional understanding of land. Farmers should be made more aware of the forces that have brought land about, and by which it is maintained. They need to know the reasons for doing the necessary things. They need to have what I like to call a biological concept of land. How futile it is to passively follow a recipe without understanding the mechanisms behind it. People must be able to see and comprehend some of the relations between organisms and their environment—an ecologic view.

Ecology can be made the fusion point of all the natural sciences. Agriculturists and conservationists, especially, should have a mind that can take a biotic view of land. They must not conceive land as merely soil, but to include man, plants, animals, air, and water. The biotic function and economic utility of plants and animals must be understood.

A graphic picture of soil and the living things on it (the biota) is the biotic pyramid as used by the ecologists. Living things are represented by the layers of a pyramid, the base of which is soil. A plant layer rests on the soil, an insect layer on the plants, a layer of insect-eating organisms next, and so on up through various groups of fish, reptiles, birds, and mammals. At the top are the predators. Plants absorb energy from the sun. This energy flows through this biotic circuit. By such a process our land was formed. In our agricultural enterprises we have removed the pyramid which nature constructed through natural evolution. In its place we have erected a pyramid of domestic plants and animals. This pyramidal image of land conveys the idea (a) that land is not only soil, and (b) that man-made changes are of a different order than evolutionary changes and have effects more comprehensive than is intended or foreseen.

A biotic view of land enables one to see and appreciate the forces that bring it about and by which its productive capacity

is maintained. Such a view makes one conscious of the place of plants and animals in maintaining soil health. An understanding of the pyramid of life is bound to counteract our complacence, our smugness, and our apathy concerning the affairs of the land. It shows just exactly why some things must not be countenanced if land is to remain healthy.

In conclusion, let me say that the public generally must be made cognizant of the problems—must be made thoroughly aware of them—must have a clear and comprehensive appreciation of them. We all need to re-discover land; understand its functions. We need more of a reverence for it. We need to learn to love it. Only when enough people will have an enlightened awakening will we have soil conservation. It is not a separate entity. It is a by-product of a way of living—of human conduct. Its future will depend on the collective experience, judgment, and moral sense of all who have an interest at stake in it. That is everybody. And certainly the schools have a part to perform. They must teach and exemplify the things that should become a part of our *mores*. It is heartening to note the progress that has been made in conservation thinking during the last decade. Our striving is taking on more significance.

ATOMIC PARTICLES UNSEEN BY HUMAN EYE ARE MAGNIFIED 180,000 TIMES

Success in magnifying an infinitesimal particle of atomic structure to a size more than 180,000 times greater than the original specimen—an achievement opening to scientists and disease fighters heretofore unseen realms for research and exploration—was disclosed by the Radio Corporation of America to members of the Electron Microscope Society of America meeting at Princeton University.

Dr. James Hillier, one of the electron microscopy pioneers of RCA Laboratories, told of developments which have almost doubled the previous accepted bounds of magnification by electronic means. These include design of an electronic "gun" which increases the intensity of the image twenty-fold and improves illumination to such an extent it is possible to use a new telescopic viewing device. Dr. Hillier stated that an improved lens also has increased resolving power, and described how these modifications not only have enabled an operator to examine the final image visually at unprecedented levels of magnification but have made possible photographic exposures as well.

Perry C. Smith, design engineer of the RCA Victor Division, Camden, N. J., presented a paper describing new accessories of the electron microscope which will be built by his company. Among them are a newly designed vacuum unit, a new vacuum gauge, and an electron spray device for use in diffraction studies. He also described an improved method for fabricating films that support specimens to be viewed in electron microscopes.

IS BIOLOGY A SCIENCE OR A FAD?

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This question is not asked facetiously. It is asked in all sincerity and with full knowledge that there will be both favorable and unfavorable reactions. It is a question, however, which is in the minds of many outstanding teachers in the field and a question which must be answered for those who are instrumental in selecting or preparing courses of study, texts, and study guides.

It might be well at this point to clarify the terminology used in the title of this article. Any standard dictionary defines *science* as a knowledge which is coordinated, arranged and systematized with reference to general truths or laws. A *fad* is a passing notion, custom, or style followed enthusiastically for a while; a pet idea; or a hobby.

If biology is a science, it should be based upon fundamental truths or laws. Laws of science are universal laws. They are the same in Pittsburgh as they are in Pumpkin Center, London, or Madrid. They are as sound today as they were in the past. The laws in the field of living things are as basic and unchanging as those in the field of chemistry and physics. It is only the applications of these laws which change with new developments, environmental, and social changes; yet an analysis of textbooks and courses of study reveals a wide range of opinions in the selection of subject matter and the degree of emphasis given to the different areas chosen. There are those which ignore the basic fundamental truths or principles. In some there is evidence of an honest attempt to use them as a framework around which to build the subject matter. There seems to be no agreement, however, among those which contained basic truths as to what truths were basic. There are still those which reveal an organized body of factual material gathered around a few selected topics such as ecology, taxonomy, heredity, etc.

In an attempt to meet the needs of the students and the community, there are those groups who have built the biology program around conservation; others who would make it "Biology for the Consumer"; those who would make it a reading course, a health program, or a course in social problems.

Some have advocated that the terminology should be deleted. Chlorophyll would be "green coloring matter" and the stomata

would be "breathing pores." The vocabulary of biology would be omitted entirely. There are those who would classify the plants and animals into their different groups without any basis for classification. They would avoid mention of the less familiar groups. Only the most obvious structures and processes would be mentioned. There are those areas of the country where evolution is not mentioned and where sex is common only to frogs and earthworms.

Protoplasm is the basic substance of which all living things are made. It is the physical nature of this colloidal substance which makes it possible to carry on all the life processes and activities of living things; yet protoplasm is only a word in most text books. In some there is a lengthy explanation on its chemical structure with a discussion of elements and molecules accompanied by laboratory experiments with free nitrogen, oxygen, and hydrogen. All this in spite of the fact that none of these elements are found free in protoplasm. In many instances neither the student or the teacher have had any previous knowledge of chemistry.

These are not idle accusations. They are based upon the experience of those who have been in position to observe these practices over wide areas, those who have made a careful analysis of the contents of biology texts and courses of study, those who have made studies of questionnaires which represent the opinions of large numbers of biology teachers selected upon the basis of their position and experience.

One finds himself lost in an attempt to determine how all this came to be. It is not likely that anyone knows all the answers, but it is quite evident that there is missing in the field of biology a unity of purpose. Many biology teachers are satisfied to follow a book, a study guide, or a personal interest. Others have taught the subject for many years; they belong to the "old school" and are satisfied to continue in the groove. Some of the more interested, progressive, conscientious teachers have struggled in vain to find a way.

You will wonder why all this concern when statistics show that biology is still a popular subject among high school students. This statistical popularity is based upon the number of students who elect biology in preference to the other sciences. The answer to this is obvious. Most high schools require one credit in science for graduation. Since biology has a reputation for being easier than chemistry or physics, it is natural that it

should be the one most frequently chosen. Moreover, biology is generally offered in the freshman and sophomore years while chemistry and physics are left for the junior and senior years when the greatest number of students begin to drop out of school.

If biology is to be the once science which reaches the greatest number of students, it is all the more reason why the subject should be sound in content and purpose; and it should be taught by teachers who are adequately prepared to teach it.

The time has come when leaders in the field, outstanding classroom teachers, and organizations of biology teachers should get together and prepare a definite workable program for the teaching of biology as a science. This program should be made available to those institutions which prepare high school biology teachers, those who prepare biology texts, study guides, and courses of study.

Such a framework of basic fundamentals would in no way place any limitations on the initiative of the teacher in developing them to meet the needs of the student and the community in which the subject is taught. It would place no limitations on the amount and choice of subject matter used to clarify their meaning.

The need for biology *as a science* has never been greater than it is today. An intelligent understanding of the rules of the game of life and the laws or principles in accordance with which the phenomena of life proceeds is urgently needed to point the way to health, worthy achievement, and cooperation with our fellowmen.

RECENT PROGRESS IN THE BIOLOGICAL SCIENCES

Several powerful new pesticides, restricted to military and experimental uses during the war, were released for civilian employment; they include DDT and Gammexane (British) against insects, 1080 and ANTU against rats, 2-4-D and ammonium sulfamate against weeds, and G-412 and G-410 specifically against ragweed.

Germ-stopping substances similar in action to penicillin were found in lichens, wilt-resistant tomato plants, leaves of Scotch thistle, mullein and peony, and the fruits of blueberry, currant, mountain-ash and honeysuckle.

Formulae for several effective mosquito repellents were released by the Army and Navy.

Heartbeats of birds, many times more rapid than those of humans, were countered with a sensitive electrical instrument attached to the twig on which the bird perched or even under the nest.

Plant disease viruses, far too small to be seen with any instrument were studied by depositing gold films, eight Angstroms thick, on protein particles of submicroscopic size and using an electron microscope.

HISTORY OF ARITHMETIC*

I

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The first cave man that found it necessary to take notice of the difference between one animal and more than one animal, or one enemy and more than one, started the study of arithmetic. And that other cave man in the next cave who had to take notice of the difference between a big deer and a little one made another contribution to the subject, for size as well as number enters into arithmetic. Rather a simple beginning to the subject of arithmetic, wasn't it?

Once one of those primitive men saw several enemies on a hill and crept back to his cave to warn the men who were his friends. Now, we'll say he and his friends were six in number, but of course they had no idea what six meant and had no word for six. They had no use for numbers and they couldn't count. But it was important that these men know whether they could drive away the enemies or whether they ought to hide in the caves. They did not know it, but the problem that they faced was connected with what the mathematician calls *one to one* correspondence. If for each cave man there was one enemy, then a battle was in order. But if there were two or three enemies for each cave man, then it would be best to hide. That is when the subject of arithmetic took a long step forward. The cave man who was reporting by grunts and groans began to put up his fingers one by one, grunting his word for enemy each time he put up a finger. Perhaps a little different kind of grunt went along with the different fingers. That was the beginning of counting, and the fact that the cave man had ten fingers explains why number systems all over the world are almost always based upon tens. Ten ones make ten, ten tens make one hundred, ten hundreds make a thousand, and so on.

How many enemies? How many friends? How many days?

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* The articles of this series are from scripts of broadcasts over radio station WBAA. This series is a part of the Purdue University *School of the Air*. There is one broadcast each week on some subject of mathematics. The first series had the general title *Mathematics Is Where You Find It* and ran for thirteen weeks. A second series of thirteen broadcasts has the title *Mathematical History and the Men Who Made It*. The articles as here given are in the exact words in which they were broadcast. They have been timed to be read in thirteen and a half to fourteen and a half minutes, at the rate found most effective for understanding by the radio audience. Rebroadcasting of these talks in the original form or with adaptations is permitted. Write to the author for specific permission for their use.

How many pigs? How many children? How many slaves? How many wives? How much wheat? How many robes? How much land? How high? How wide? How far? How many ships? How many pieces of money? How many war chariots? How many dollars? How many airplanes? How many radios? How much electricity? How many molecules? From the earliest times down to our own and on into the centuries yet to come these and like questions are asked over and over, and the answers are always in terms of arithmetic. For thousands of years men have been adding to and changing the systems of arithmetic to make them serve the needs of people better. I shall call your attention to some of the steps in this long history.

When the peoples of the world began to become civilized they found need for arithmetic, among other things. They needed to know how to figure the value of things bought, sold, grown and lost. They needed to be able to figure the amount of grain in a bin or the value of jewels. All this meant they had to know many of the things you and I learn today: how to add, subtract, multiply and divide and other such operations. We would not suppose that they did these things as we do them today.

One of the differences between ancient ways of doing arithmetic and our own is that they did not usually put down numbers as we do and add or multiply them on paper; they did most of their figuring with pebbles or shells on rows marked in sand, or with wooden beads strung on sticks or wires in a frame, and when the answer had been found in this way, then wrote it down. The device with the pebbles or beads was called an abacus. That is A-B-A-C-U-S. The Romans and Greeks used it nearly 3000 years ago, the Arabs and Hindus used it, the Chinese and Japanese used it, the Russians and Mongols used it; in fact it was used all over the civilized world. And from the time it was first used, its use has increased. It is still used in Japan, in China, in Russia, in Turkey and in other places. It is the world's oldest adding machine.

Many people first used the letters of their alphabet to express numbers. The Greeks did this, the Jews did it, and the Romans did it. No doubt you know something about the way the Romans did this. X was 10, V was 5, C was 100, and so on. You may have wondered how the Romans couid possibly add or multiply with such an inconvenient system of numbers. Well, they would have found it as hard as you do, but remember all

the work was done on the abacus and the letters were used only to write down the answers.

But there were other people who used special written symbols with special spoken names to express their numbers. Among these were the Hindus and the Arabs who were close neighbors and often borrowed ideas from each other and from other neighbors. The system of numbers which we use came to us from the Arabs who got part of it from the Hindus and added some ideas of their own. There are three things that make it the most perfect number system ever invented. First, there is the idea of place value; that is, a 7 is 7, or 70, or 700 depending upon where it is placed. Second, there is a figure, the zero, which shows that there is no value where it is written. Third, the figures are different from the letters of the alphabet so they always have the same meaning instead of doing double duty as letters and numbers. If you add to these facts our system of figuring directly from written numbers without using a device like an abacus, you will see that we have a number system about as perfect as it is possible to make it.

How long did it take to perfect this system? Truly, we do not know, but we do know that its development can be traced back at least 4000 years. The Arabs brought it in its present form to Europe about 1000 years ago, and from Europe it came to us.

One of the things that is necessary in doing number problems without an abacus or some such device is to learn by memory a lot of simple number combinations such as adding and multiplying results. When you have a 7 and a 9 to be added, you have to know right off that the sum is 16 without having to count fingers or dots on the blackboard, and when you want 6 times 8, you have to know at once that the result is 48 and not have to put down 8 six times and add. Now, if you don't know these number combinations, there is no better time to start learning them than now. Sit down this evening and do a hundred or so of these number combinations with someone to write or say problems for you. Ease of adding, subtracting, multiplying and dividing depends on knowing them. Learning these number combinations has been one of the tasks of school pupils for hundreds of years, but it has been done in lots of different ways. When I was a pupil in school, we had to learn and recite what we called the multiplication table which ran from 1 times 1 to 12 times 12. Before that, pupils often had to learn the tables up

to 24 times 24. Perhaps you do not learn the numbers in any such formal way today, but in some manner you learn at least as far as 9 times 9 so that you know the results readily.

Today we generally speak of the *four* operations of arithmetic and we mean adding, subtracting, multiplying and dividing. Six or seven hundred years ago there were seven operations instead of four. Besides those you and I have to learn, pupils of that time had to learn duplation (that is D-U-P-L-A-T-I-O-N), mediation (spelled M-E-D-I-A-T-I-O-N) and root extraction. At some time or other you still learn to find the square root of a number but not cube root. Duplation meant doubling numbers, or multiplying by 2. For many hundreds of years doubling and redoubling has been used instead of straight multiplication. You might try multiplying, say 17 by 13, just by doubling and adding. It is quite easy but where the process is much used, special rules help get quick results.

Mediation meant dividing a number by 2, then that result by 2, and so on as long as necessary to get the result of *any* required division. It was the opposite of duplation. Often duplation and mediation were combined as in the method of multiplication which we sometimes today call the Russian peasants' method. This method makes it possible to do any multiplication simply by multiplying and dividing by 2 in a certain way. It is very easy but takes more time and more paper than the way you know.

I have mentioned the abacus as an instrument used all over the world for doing arithmetic. I shall tell you briefly of a few other aids to number work used in different times and different countries. Of course, all over the world, people have long used their fingers as aids in figuring. That explains why 10 is the basis of nearly all number systems. Some uncivilized peoples also used their toes as aids to number combinations.

Another device was what were called Napier's rods. These were the invention of a Scotchman named John Napier who also invented logarithms. Each rod or piece of bone had the multiples of one number on it. To multiply, say by 245, one would place side by side the rod with 2 at the top, the one with 4 at the top and the one with 5. By combining numbers found on the rods, the answer was quickly found. These rods were carried by pupils and used in school. Business men used them, too. What do you suppose these people did when they forgot or lost their rods? Maybe the fact that they were not always at hand when needed explains why they passed away long ago.

Just three hundred years ago, a priest by the name of Ciermans suggested the first adding machine with levers and wheels as they are now made, but he did not actually make such a machine. The first such machine actually made and patented was made by a Frenchman whose name was Pascal. The machine was not a success. Many others were tried but with little success. Just over one hundred years ago an Englishman by the name of Babbage made a machine that was better than any up to that time but even it was scarcely successful. The good machines which you now see in stores and banks were first made about 35 or 40 years ago.

Every college student taking engineering carries an arithmetic device called a slide rule. It has been in use for nearly 300 years and saves so much work that we shall probably go on using it for a long time to come.

One of the subjects that has always caused trouble is that of fractions, and of course there was much more trouble with them in ancient times than now. Four thousand years ago, the Babylonians knew something about tenths and hundredths and other fractions which we call decimals, although they did not use decimal points as we do. Halves, thirds and other common fractions gave ancient people a lot of trouble, especially when there was a fraction whose numerator was more than 1, such as $\frac{2}{3}$, 7/11 and so on. They had special ways of changing all such fractions into other fractions having 1 as numerator. For example, an Egyptian by the name of Ahmes (that is A-H-M-E-S) wrote a book 3500 years ago in which he shows that $2/42 = 1/42 + 1/86 + 1/129 + 1/301$. How he and others of his day found this out, we do not know. They called these unit fractions because all numerators are 1.

In many ancient countries they wrote many fractions as sixtieths, or sixtieths of sixtieths and so on. We call such fractions sexagessimals. We still use them in measuring time and angles. Three minutes means three sixtieths of an hour or of a degree, for example, and it has been this way for thousands of years. How men ever got started using sexagessimals is a mystery. It would have been much better if they had used hundredths rather than sixtieths, but it is a little late to change now.

We are indebted to the Arabs and Hindus for our method of writing common fractions. They wrote them as we do 800 years ago.

"DROP OUT" GUIDE LINES

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Construction of charts, such as the "target" wind frequency chart of Fig. 1, is usually a laborious process. In many instances, guide lines are drawn in pencil, and erased after the wind vectors are inked. Where many copies of a chart must be made, some with guide lines, and some without, drafting time is excessive.

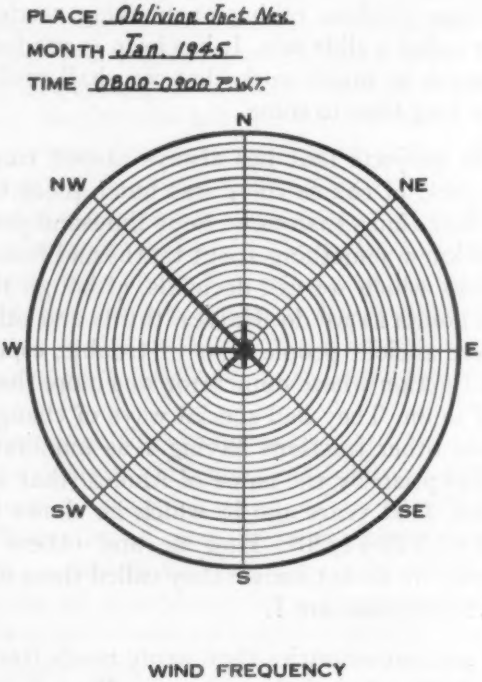

FIG. 1. "Target" wind frequency chart, showing guide lines as a full tone. Location, for security reasons, is apocryphal.

If the lines desired in all copies of a chart are printed in black, and guide lines, or lines wanted only in some copies, are printed in either red or blue, the guide lines are easily visible to the draftsman, and may be altered in tone, or eliminated completely, by use of simple, standard, photographic processes.

Results with these processes are shown in Figures 1, 2, and 3.

In Fig. 1, if the guide lines are blue, copying is done with a red filter and supercontrast panchromatic film (such as panchromatic Kodalith). If the lines are red, a blue filter is used with the same film. This figure can be printed as a line cut (the cheapest method).

In Fig. 2, where the guide lines are half tones, supercontrast panchromatic film without a filter is used with either red or blue guide lines. In many instances, lithographer's ortho film (such as Kodalith Ortho) will also give satisfactory results.

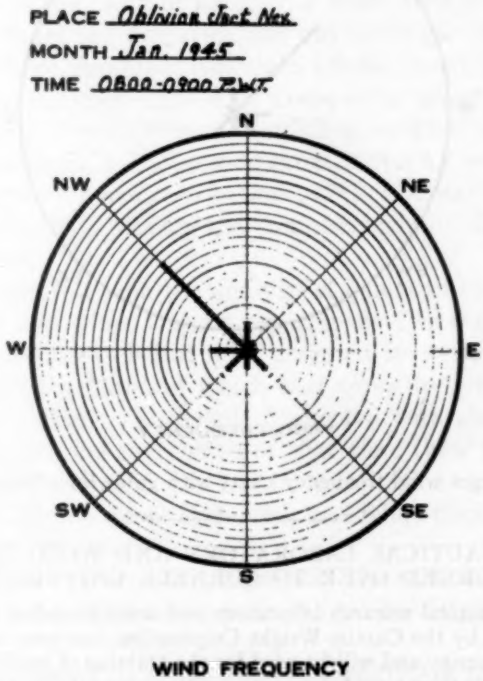

FIG. 2. Target wind frequency chart with guide lines as half tones.

In Fig. 3, in which the guide lines are "dropped out," copying was done on supercontrast panchromatic film with a filter of the same color as the guide lines. Blue lines may also be dropped out by copying on "color blind" film without a filter. This figure may be reproduced as a line cut.

Proper choice of ink is important, but most printers are now aware of the properties of their inks, and will supply the right colors if the problem is explained to them. Wider use of "drop

out" guide lines should result in a considerable saving of time and money in the preparation of research reports.

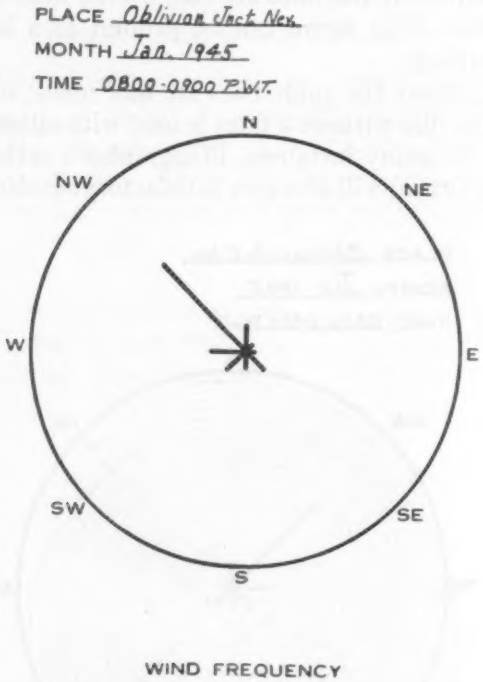

FIG. 3. Target wind frequency chart with guide lines "dropped out."

AERONAUTICAL LABORATORY AND WIND TUNNEL TURNED OVER TO CORNELL UNIVERSITY

The aeronautical research laboratory and wind tunnel at Buffalo, built and operated by the Curtiss-Wright Corporation, has been turned over to Cornell University and will be used for the training of graduate students, who will divide their time between the engineering school of the university proper and this laboratory. The Buffalo facilities will be supported by a number of leading Eastern aircraft manufacturers.

The laboratory, built in 1942, contains the most modern scientific equip- and testing devices known to aeronautical research. It includes also well-equipped chemistry, physics, hydraulic and electrical laboratories, a model shop and a technical library. Its wind tunnel, however, is its most outstanding equipment.

In this wind tunnel scale airplane models can be tested in air velocities in the speed-of-sound range, under varying pressure conditions. Also there are miniature wind tunnels where air travels at supersonic speeds, and one of the world's largest altitude chambers, where conditions of pressure, temperature and humidity up to 80,000 feet can be reproduced.

Dr. C. C. Furnas, who has headed the laboratory since 1943, will remain to direct its activities for Cornell.

THE MATHEMATICS OF GAMBLING*

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First, let me assure you that my interest in games of chance is purely academic, and not professional. Secondly, I would like to say that I am pleased to see such great interest in mathematics. Even though a survey by Dr. George Gallup showed that 45% of the adult population of the United States indulges in one form of gambling or another, I know that your interest is primarily in the mathematics, and not in the gambling.

Of the 45% of gambling American adults, many persons engage in several types. Including those who wager in several ways, the most favored forms of gambling reported were: bingo, raffles, and lotteries, 24%; playing cards or dice for money, 20%; betting on athletic events or elections, 17%; slot machines, 16%; punch-boards, 15%; playing the "numbers game," 7%; betting on horse races, 7%.

I became interested in the analysis of games of chance about fifteen years ago when I was a reporter on *The Daily News* in Passaic, New Jersey. I had been assigned to write a series of news stories on a carnival which had come to town. I spent several days talking with people connected with the show and the carnival. During the course of that time these people told me—for my own benefit, and not for publication—exactly how many of the games were conducted, and what tricks were employed by the operators.

The study of mathematics, especially the sections in algebra on permutations and combinations and probability, heightened my interest.

There are many thousands of games and more thousands of variations of these games. For that reason it is practically impossible for any one person to analyze every game. I have attempted to analyze those which are most common and most widely known, and have assembled them in a book titled *You Can't Win*, to be published soon.

There are two ways of analyzing a gambling game. The mechanics of the analysis involve, in general, the use of elementary algebra, elementary theory of probability, and elementary statistics.

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* Presented at the University Training Center, U. S. Army, at Florence, Italy, on October 30, 1945, as one of the series of weekly lectures.

The first method is known as *a priori* probability, sometimes called pre-game probability. This is simply an examination of the rules of the game, the computation of the total number of all possible choices, and the possible number of successes. *A priori* probability depends only on a hypothetical game and mental gymnastics.

The second method, called *a posteriori* probability or post-game probability, actually uses the results observed in a regular game or contest.

Without going into a technical discussion of probability, let me define it simply as the ratio of the number of successful events to the number of events, $P_s = s/s+f$, where s is the number of successes and f is the number of failures.

We shall be concerned here only with *a priori* probability, the first type. Over a long period of time and a large number of plays, the results obtained from actual games are approximately equal to those obtained by the pre-game analysis. Since it is more important that you know something about the odds in a game before you play it, the *a priori* probability should be your guide.

There are two simple laws of probability that you should know. First, that concerning *either/or* probability. Suppose you consider a single die (singular for dice). It has six numbers on it. There is only one No. 6 on it. The probability that a six turns up in a single toss is therefore one-sixth. Now what is the probability that a five turns up? It also is one-sixth. If you ask what is the probability of getting *either* a six *or* a five in a single toss, the answer is one-sixth plus one-sixth, or two-sixths. So, in *either/or* probability we add the probabilities of the individual events.

The other important law concerns *this-and-that* probability, sometimes referred to as *both/and* probability. Suppose I ask the question: what is the probability of getting a six on the first toss of the single die and then getting a five on the second toss? Here we multiply the individual probabilities and obtain one-sixth times one-sixth, or 1/36.

One definition, of which we shall make considerable use, is that of *expected number of occurrences*, $E = Np$, where N is the total number of trials and p is the probability of success.

Summarizing, we have: in *either/or* probability, add the individual probabilities; in *this-and-that* probability, multiply the individual probabilities; *expected number of occurrences* $= Np$.

Now we are ready for the analysis of games:

1. *The Numbers Game.* This game is very common in large cities. It is conducted by an operating "ring" which determines the method of selecting the number to be declared the winner for a day. The "ring" permits you to bet any amount on any number from 0 to 999, giving you a choice of 1000 numbers in all. Generally, the winning number is determined by the units digits of pay-off prices at some race track.

If you are unfortunate enough not to have placed a bet on the winning number (and this is nearly always the case), you lose whatever you have wagered on other numbers. On the other hand, if you selected the winning number, the "ring" pays you either 500 for 1 or 600 for 1, depending upon the location of the game. The New York City area recently went on a 500 for 1 basis.

Thus, if you have wagered one dollar on number 654 and that number was the winner, you would receive $500. There is an unwritten law that the bookie (the fellow who took your bet) receives 10 per cent of the winnings, so you have to give him $50, which leaves you $450.

This sounds like big money, but let us examine the game. If you had selected each number, you would have wagered $1000 in all. Since only one number can win, you receive $500 in return, of which you give $50 to the bookie. You actually get a return of $450, while the "ring" and the bookie pocket $550.

This simply shows that only 45 per cent of all the money wagered is returned to the bettors while the "ring" and the bookie grow fat on the remaining 55 per cent.

2. *Roulette.* This game is based on a round wheel, divided into thirty-eight sectors, eighteen of which are red, eighteen black, and two green. The wheel is mounted horizontally and spun about its axis. The sectors are numbered 0, 00, 1 through 36. Depending upon the rules set up by the "house" operating the game, wagers may be made upon certain numbers.

Let us consider the first possibility. Players are permitted to bet on any number from 1 through 36, or on the colors red or black. The house pays off at 35 to 1 for each number winner, and even money for each color winner. The 0 and 00 (green numbers) are called the "house" numbers. If the spin of the wheel is such that the little ball falls into the 0 or 00 sectors, all players lose their bets. This gives the house 2 chances in 38 of winning all the money. If one dollar is wagered on each number

each time, then the house loses nothing on thirty-six of the spins, and wins all $36 on two of the spins. This actually means that the house takes 5.26 per cent of all the money wagered.

One of the boys in the art department, who helped make up the posters for this lecture and who has been around a little more than I have, told me that there were places where the pay-off odds were 36 to 1, and that the players were permitted to bet on 0 and 00. In this case there are 38 possible choices, and if we analyze the wagering, we see that with the board full each time, the operator would be taking one dollar out of every spin. This amounts to 1/38 of all the money wagered, or approximately 2.6 per cent.

However, I think that there may be some confusion about 36 to 1 and 36 for 1. The first, 36 to 1, means that if you bet a dollar, you get back $37, your own plus the thirty-six. The second, 36 for 1, means that you actually get $36, which includes the one you wagered.

This analysis shows that the house takes a percentage of all money wagered. All computations have been made on the assumption that the game is run legitimately. Some houses have brakes on their wheels, similar to automobile mechanical brakes, which may be operated by the croupier at will.

Other houses have wheels fitted with small switches which may be operated at will to make certain sectors narrower than others and thus prevent the little ball from falling into a sector on which number there is an extremely heavy play. Still others have a system of magnets fitted into the wheels, so controlled by the hand of the operator that a metal ball will be drawn into a sector on which there is little or no play.

3. Double-Your-Money-Quick. This is a dice game, found any place that is frequented by dice players. Sometimes the game is called Chuck-a-luck. Three dice are tossed by the operator, usually in a small metal cage. A large cardboard, with numbers 1, 2, 3, 4, 5, and 6 is placed on a table. Players may place wagers on any of these six numbers.

If a player bets one dollar on No. 1, and the dice, when tossed, read 1-2-3, then the operator gives the player on No. 1 a dollar. If there are players on No. 2 and also on No. 3, the operator gives these players a dollar each. He scoops up the dollars wagered on 4, 5, and 6.

When all six numbers are covered by a dollar each, and the operator tosses three different numbers, he simply picks up the

three dollars on the three losing numbers and turns them over to the players on the three winning numbers. The operator does not win or lose in this case, called singles.

However, when the dice are tossed 1-1-2, the operator gives two dollars to the player on No. 1, and one dollar to the player on No. 2. He picks up the dollars on 3, 4, 5, and 6, which leaves him a profit of one dollar on the toss.

If the dice turn up 1-1-1, the operator pays the winner three dollars, and scoops up the five dollars on the other five numbers. He makes a profit of two dollars on such a toss.

Thus, it is clear that the operator makes one dollar when doubles turn up, and two dollars when triples turn up.

Since the expected number of occurrences of doubles is 90 times in 216, and that of triples is 6 times in 216, the operator makes $102 in 216 tosses. Thus, he takes $102 out of $1296 wagered, or a profit of 7.87 per cent of all money wagered.

4. Double or Nothing. In this game the persons taking part are permitted to do the dice-tossing while the one who operates the game simply pays the winners and collects from the losers. The idea behind the game is whether the tosser can throw doubles or triples with three dice. If he does not toss doubles (two of a kind) or triples (three of a kind), he loses whatever he has wagered. If he does toss doubles or triples, he wins from the operator as much as he has bet, that is, he doubles his money.

Out of 216 tosses, the expected number of singles is 120, while that of doubles is 90 and that of triples 6. Thus, there are 120 losing tosses and only 96 winning tosses. A wager of one dollar on each toss would find the tosser losing $24 on every 216 tosses, or $24 out of every $216 wagered, which amounts to 11.11 per cent.

5. Over Seven, Under Seven, Exactly Seven. A board is set up with the three categories. You are permitted to bet on any one or all. Two dice are thrown and a total is determined. If you bet Over Seven, and a number over seven turns up, you get even money. If you bet Under Seven and win, you get even money. If you bet Exactly Seven, and a seven is tossed, you get 2 to 1.

Examining the probabilities, we find that there are 15 chances in 36 for Under Seven, 15 in 36 for Over Seven, and 6 for Exactly Seven. Hence, the true odds are 21 to 15, 21 to 15, and 5 to 1, respectively, and these are the odds that the operator should pay in an even (legitimate) game.

Suppose wagers were made on all three items thirty-six times.

The expected number of Under Sevens is 15, and so you would get back $30. You would get a like amount for the 15 Over Sevens. On the Exactly Sevens, appearing six times, you would get back $18. The total return from the $108 wagered is only $78, which gives the operator of the game a profit of $30 out of $108, or 27.7 per cent.

6. Regular Dice. While we're discussing the subject of dice, let us look over the odds in a regular game. Most of you are familiar with the rules. Now suppose you have made a roll and have a point to make in order to win your bet, what is the probability of making that point? From the individual probabilities shown on the chart, it is clear that the odds against making a No. 4 are 2 to 1, against a No. 5, 3 to 2, against a No. 6, 6 to 5, and similarly respectively for the points 10, 9, and 8.

You might say, "Well, suppose I just sit around and bet each time against a tosser whose point is 6 or 8, this bet at even money." It is true that the odds are in your favor, and you should expect to win 6 out of every 11 wagers, making in the long run a profit of $1 on $11 wagered, or approximately 9 per cent. However, the other boys in the game soon get wise to this and refuse to bet with you.

Suppose then you play just as the others do, and take your turn tossing the dice. Let us look at the odds against a dice-tosser. In 1980 tosses, by *a priori* probability, the expected wins and losses are:

		You Win	*You Lose*
If First Toss is	2	0	55
" " " "	3	0	110
	4	55	110
	5	88	132
	6	125	150
	7	330	0
	8	125	150
	9	88	132
	10	55	110
	11	110	0
	12	0	55
Totals		976	1004

The odds against the dice-tosser are 1004 to 976, or 251 to 244. This means that in 1980 tosses, the expected number of

wins is 976 and expected number of losses 1004. At a dollar each, the dice-tosser loses $28 out of $1980, or 1.41%.

To all appearances this may seem to many to be an even game. However, it is not. The house or the operator of the game has not yet been considered. The house takes a fixed percentage of all money wagered in the game, sometimes 3 per cent, sometimes 4, sometimes 5. Or the operator of the game who acts as the judge and handles the money takes a "cut" from the money on the table after every two "passes," which means that if the tosser wins twice in a row, some of his money goes to the operator. This "cut" may be a dollar, or two, or five.

The operator does this all during the game and at the end of the evening he will have taken a couple of hundred dollars out of the game.

Quite some time ago on one of my first visits to the Rest Center, I saw such a game in progress. I thought the boys were playing for cigar coupons, but they turned out to be 500-lire and 1000-lire notes. The operator of the game was having a merry time for himself, calling out the numbers and holding the wagers. Every now and then, when a tosser made two "passes" in a row, the operator would pocket one of the 500-lire notes or even a 1000-lire note. And when you remember that 500-lire is equivalent to five dollars, you can well imagine how much the operator enjoyed that game.

Large gambling houses have their own rules. Every one is permitted to toss the dice, but the house never does. The house is sure of approximately 1½ per cent of all money wagered, and by adding a fixed percentage cut of all money wagered, the house "Take" runs up to five or six per cent.

In many games the dice are not true dice. They may have been constructed to toss certain numbers more often than others, for example, 2, and 3, and 12, which are losing numbers when they turn up on the first toss for a dice-thrower. Other dice are made so that they toss only sevens and elevens. Still others are made so that they toss only points, 4, 5, 6, 8, 9, 10, and never sevens. Clever manipulators can interchange these dice whenever they please.

Clever gyp artists generally band together in groups of threes or fours in a game, and so are able to cover up well when the switch in dice is made. Or when a "sucker" reaches for the dice to see whether they are phonies, one or two of the group will pick up the dice first and assure the "sucker," after a phoney examination, that the dice are true.

In some places the dice tables are wired so that they are magnetized, and with the current switched on, magnetic dice may be made to turn up a 7 when the operator wants to "seven out" a tosser.

7. *Match a Card*. A betting board is set up, with the cards Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King shown. The object of the game is to call a matching of the number of a card, not the suit. Players may wager on any of the thirteen cards.

A card from a deck is turned up. If you have called it, you get 10 to 1, that is, you win $10 by betting $1. If you have not called it, you lose. Examining the possibilities, you can see that there are four cards of each number in a standard deck, hence selecting a single card of certain grade means a probability of 4 chances in 52, or 1/13. This means that you should have had odds of 12 to 1.

If you had covered all thirteen cards, you would have wagered $13 and received in return only $11, you own dollar plus the $10 you won on the single winner. This means that you lose $2 out of $13, or 15.38 per cent of everything wagered. Thus, the operator of the game grows rich on the more than 15% "take."

Of course, the cards must be shuffled after each selection has been made, or the probabilities change, since you have already seen the exposed card or cards.

8. *Over Seven, Under Seven*. In this game you may wager that the card turned up is either Over Seven or Under Seven. If a Seven turns up, the operator of the game wins all bets. He has 1 chance in 13 of winning. If you bet Under Seven and a No. 5 turns up, you win as much as you have bet. If, on the other hand, a 10 turns up, you lose. Actually, the operator takes 7.69 per cent of all the money wagered. Cards are generally reshuffled after a Seven appears.

9. *Straight Poker*. What are the probabilities of making certain types of hands in a straight poker game, five cards composing a hand? The chart below shows that there are 2,598,960 different hands, of which certain numbers are royal flushes, straight flushes, fours-of-a-kind, full houses, etc.

Notice the tremendous odds against getting a certain value hand.

I will not go into detail concerning the various types of draw poker and other card games since each game involves certain rules and therefore makes individual computation of the probabilities necessary.

Hand	*Number*	*Odds Against Getting*
Royal Flush	4	649,739 to 1
Straight Flush	36	72,192 to 1
Four of a Kind	624	4,164 to 1
Full House	3,744	693 to 1
Flush	5,108	508 to 1
Straight	10,200	254 to 1
Three of a Kind	54,912	47 to 1
Two Pair	123,552	20 to 1
One Pair	1,098,240	7 to 3
No Pair (Bust)	1,302,540	
Total	2,598,960	

10. Bridge Hands. In order to make the bridge players feel at home, let us look at the probability of getting thirteen spades. Dealing a hand of 13 cards from 52, the probability of getting 13 spades is less than 1 chance in 13 trillion. This is equivalent to saying that if every man, woman, and child in the United States were to play one hundred hands of bridge each evening, it would take 1000 days—almost three years—for the 13-spade hand to appear.

There are two fundamental rules concerning card playing.

1. Never play cards with strangers.
2. Never play cards for money.

Don't be like old Canada Bill, a well-known gambler of the gold-rush days. One night Canada Bill and a friend were in a small western town. Bill disappeared and his friend spent an hour trying to find him. He finally located Bill in a card room back of a barber shop. Bill was playing faro. The friend pulled Bill's arm and told him to quit, the game was crooked. But Bill replied, "I know it's crooked, but it's the only one in town."

Many gamblers and sharpers use marked cards. Such cards may be purchased from several well-known playing card companies. Other decks of cards may be marked by professionals. The cost of a deck of well-marked cards is generally around ten dollars. Crude sets sell for one or two dollars.

While I was teaching at the University of Illinois, I happened to stop at a small shop one day. Conversation with the proprietor disclosed the fact that he had once been a card-player. He showed me a deck of marked cards, and pointed out the marking scheme.

Cards are marked in the upper left-hand corner and the lower right-hand corner. There must be a fancy design in those corners as well as elsewhere on the backs of the cards so that the marking will not be too discernible.

The test for marked cards is to riffle the deck several times, looking at the upper left-hand corners of the cards as they move by. If the design seems to move or change, just as children's play-movie cards do, then the deck has been marked in some way. Generally, omission of sections of a small floral design indicates the value of the card.

Horse Racing. Let me read you some pertinent remarks from my book *Racing—The Sport of Kings.*

As for horse racing, it is not a game of chance. The elements of chance inherent in games of cards, dice and wheels—random selection from a group of equally likely choices—are not present at the races.

Instead, the elements of skill and physical fitness play the leading roles in racing. The physical condition of the horse, the distance of the race, the weight carried by the horse, and the jockey's ride—these are the determining factors. However, it takes more than a mere perusal of the daily paper to understand horse racing and the subjects associated with it.

Examine the return of money wagered at the tracks; look at the following. Pimlico, outside of Baltimore, Md., gives back 90 per cent to bettors, 2 per cent to the state, and 8 per cent to the track and horsemen.

Hollywood Park, Cal. gives back 87 per cent to the bettors, 6 per cent goes to the state, 5 per cent to the track and 2 per cent to the horsemen.

Florida tracks return 85 per cent, the track gets 7 per cent and the state 8 per cent.

Canadian racing is subject to a take of 22 per cent, which means that the bettors get back only 78 per cent of the money wagered. And for you fellows who have been playing the horses in Italy, Max Hill told me that the state takes 27 per cent of all money wagered, which means that if you bet on a favorite and he wins, you are lucky to get back your wager.

If you're a racing fan, you'll probably be like Dan Morgan, the fellow who was manager of Jack Britton, the fighter. Morgan was hailed by a friend while he was on his way to the race track. "Got any winners for today, Dan?" the friend asked. "Winners," replied Morgan, "Hell, no, all I hope is that I can break even. I need the money."

Carnival Games. Now let's examine some carnival games. *Carnival wheels,* the kind that are spun around to determine a winning number for a prize, are almost always equipped with a mechanical brake. The operator, standing at a considerable distance from the wheel, can stop the wheel at any number he chooses, simply by stepping upon a small pedal to operate the brake. With such wheels the probability of winning is very close to zero.

The Fishing Game. Here the player pays ten cents for the privilege of scooping up a small wooden fish from a circular trough of moving water. Each fish is numbered and each number designates the prize, the small numbers designating small prizes (whistles, ash-trays, trinkets), and the large numbers designating large prizes. Generally, the water trough goes around behind a curtain, where the operator's assistant removes or replaces winning fish at certain signals from the operator. Thus, while encouraging a prospect to try his luck, the operator can select a large prize winner. When the player tries, he always gets a small prize.

The Rolling Ball Game. The odds against the player in this game are about 40–1. The cost of the game is generally 50¢. The player rolls six balls down an inclined plane toward thirty-six holes. Six of these holes are marked 1, six 2, six 3, etc. The total of all the holes into which the six balls fall determines the score.

Players obtaining totals less than 13 or more than 29 are winners and are rewarded with a radio, worth about $10.

The probability of obtaining a score less than 13 or more than 29 is 40–1 roughly.

Having covered most of the ordinary games of chance, let me impress upon you that you actually have two things working against you in any gambling game: (1) the mathematical probabilities, (2) the operators of the game.

In games of skill, such as dart-throwing, ball-tossing, covering the red spot, pitching pennies or hoops, the probability of winning is very small because the player has not developed the skill necessary to perform the required act.

Constant practice over a period of time would develop that skill, but most people have neither the time nor the inclination to practice.

Considering the above information, let me state, in conclusion, in capital letters, "YOU CAN'T WIN."

TEACHING POWER IN THE SOUND FILM

JOSEPH E. DICKMAN
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Science teachers have pioneered in the use of the instructional sound film for at least two reasons: first the many abilities of this communication medium are particularly valuable in science teaching, and secondly science teachers are accustomed to handling various apparatus and had no hesitation in using projection equipment.

Let us refresh ourselves on the powers of sound film communication, remembering, of course, that education is essentially communication—communicating knowledge, attitudes and skills to the learner.

Top power in the sound film is its ability to speak with a picture language. All the words in the dictionary could not accurately duplicate the job done by the ten minute film Common Animals of the Woods. So much instruction resolves itself into mere verbalism because we forget that students do not have the background of experience which makes words meaningful. We forget that spoken words are sounds which are substituted for things and whose meanings the communicators have agreed upon. Very often the learner has not yet been in on the agreement. The written word is still further removed from reality since these little black marks are symbols for sounds which are symbols for things. Photography has enabled us to show the things themselves and obviate a lot of verbiage. The use of the sound film can greatly reduce the percentage of instructional time we spend mastering the mechanics of communication and enables us to get at the real job of communicating knowledge.

As science teachers our principal tool of visual instruction is still the blackboard but all the gestures in our repertoire will not turn our static drawings of an electromagnetic field into the expanding and collapsing lines of force so essential to a proper understanding of an electric motor or generator. The use of animated drawings in an instructional sound film such as *Electrodynamics* simplifies these difficult concepts. Just recall the gaps left in our description of the circulation of the blood which man took centuries to discover and yet how simply the animated motion picture conveys this understanding. Some people resent this very simplicity on the ground that it takes the work out of learning and removes the opportunity for rigid mental disci-

pline. Let them rest assured that even the sound film leaves all too many difficult problems for the learner.

The ability of the sound film to annihilate space and bring any part of the world instantly into the classroom is an Aladdin's lamp for the science teacher. Perhaps even the science teacher himself has never seen a volcano, or a glacier, or a coral reef, or a geyser yet there are instructional films now available to make these and many other phenomena realities for every pupil.

How many biology students with one eye on the microscope and the other on the illustration in the text have actually observed protozoa under the ideal conditions and through the finest instruments as has every student who has seen the instructional sound film *Tiny Water Animals*. How many students have the patience to watch the complete germination of a seed. How many would be foolish enough to waste that much time when time-lapse photography shows it in a few minutes in the film *Plant Growth.*

Even our best explanation of such complicated phenomena as the life cycle of the Dodder causes all but the best scholars to be lost at the turns, but with any sound film everyone is attentive to the end. This attention factor is prerequisite to learning.

Many of the fine demonstrations we have spent so much time preparing and executing have been lost to all but the pupils in the very first row. The motion picture closeup can give the pupil in the back row a better view of the demonstration than the pupil in the front row could have of the real thing, and with the desired results every time and with a fraction of the effort and preparation. Compare the motion picture demonstration of Brownian movement as found in the film *Molecular Theory of Matter* with an actual demonstration.

Think of the advantage of high speed photography in demonstrating the actual vibration of a violin string as portrayed in the film *Sound Waves and Their Sources*.

When color film is in common use, as it will be in the near future, what a job it will do not only in those subjects where color is essential as in nature study and in the science of color itself but also in other areas where color is a part of reality. Even in such simple things as line drawings the use of two or more colors makes a sizable contribution to the effectiveness of an illustration.

The mere reading of the hundreds of instructional film titles in the field of science tells a story of the unique contribution the sound film makes to modern education. To sample a few: Electrostatics, Vacuum Tubes, Mountain Building, Atmosphere and Its Circulation, Water Power, Body Defenses Against Disease, Water Birds, Seed Dispersal. Some of these subjects can be taught somewhat without the motion picture but why do it the hard way when a high powered teaching tool is at hand?

Of course there are obstacles to a wider use of classroom films such as the relatively high cost of film that comes with high production costs and limited market and then there is the still greater projector problem. In a following article the problem of projection equipment will be discussed.

BOG POLLEN DEPOSITS INDICATE ICE AGE DATE FOR FORMATION OF CAROLINA "BAYS"

The Carolinas' mysterious "bays," great elliptical depressions in the sandy coastal plain now mostly filled with bog deposits, originated during the later phases of the last great Ice Age, studies of fossil pollen carried on by Prof. Murray F. Buell of North Carolina State College here indicate.

Prof. Buell collected samples of soil from one of the bays, known as Jerome bog, taking them at six-inch intervals from the surface down through seven feet of peat and two feet of underlying clay, to the sandy soil at the bottom. Pollen grains preserved in the ancient soils were identified and counted.

Pollens from the lowermost samplings represented such trees as blackgum, native to the region today, indicating a climate not unlike that of the present when the depressions was first formed. Above this, in the bottom clay, fir pollen is predominant, together with oak and hickory. This is the kind of forest now found in northern Minnesota, where the typical mixed hardwood forest of the United States meets the southern-most extension of the Canadian evergreen forest. Nearest firs to the "bays" now grow on the tops of the Southern Appalachians, with the mixed hardwoods stopping at lower levels.

It is estimated that a mixed hardwood-and-conifer forest must have developed in the Carolinas area during the Wisconsin period which was the last great southward advance of the continental glaciers during the Pleistocene Ice Age.

The Carolina "bays," which have nothing to do with bays in the ordinary sense of the word, have been the subject of much scientific controvery. One group of geologists believe they were produced by the impacts of giant meteorites in a single catastrophic shower. Opponents claim origins from less spectacular causes, such as eddying coastal currents, outbreaks of great artesian springs and the collapse of subterranean caves producing surface depressions and sink-holes. Prof. Buell points out that if the first hypothesis is correct, the Ice Age date determined for the Jerome bog should hold for all the bays; if not, the age is valid for this bog only; ages of others will need to be determined by separate studies.

A PROBLEM IN SIMPLE INTEREST

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The fact that the + and − quantities of algebra correspond to the debits and credits of double entry bookkeeping makes it possible in many instances to replace relatively abstract algebraic reasoning by a bookkeeping "picture."

The bookkeeping point of view and the bar-graph prove to be an effective combination in the solution of the following problem.

A man borrows $250 from a bank at 7 per cent simple interest. Forty days later he makes a partial payment of $100. How much will be required to pay the balance of the note and interest at the end of 90 days?

(1) The obvious method is to compute interest on $250 for 40 days and on $150 for 50 days. (2) Bank procedure is to charge interest on the entire $250 for 90 days and then *credit* interest on the $100 for 50 days.

Arithmetic students have difficulty in seeing that the two methods are equivalent. It seems paradoxical that the bank is willing to credit 7 per cent interest on the $100 payment when only 2 per cent is paid on savings accounts.

The fact that the bank *credits* interest on the partial payment suggests bookkeeping and it is precisely in terms of debit and credit entries that the problem takes its simplest form. Fig. 1. gives a bar-graphical picture of the customer's account which corresponds to the "obvious" method.

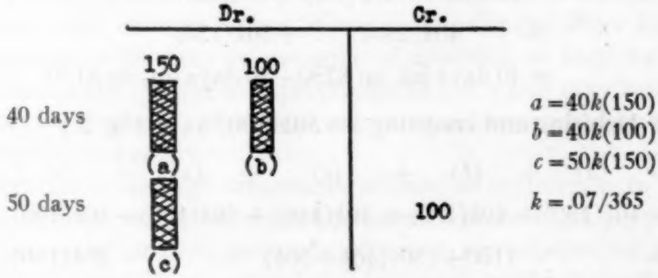

FIG. 1

A single debit of $250 would be more in the spirit of that method, but in view of the $100 payment, and to facilitate transition to the banking procedure, it is convenient to separate the debit of $250 into one of $100 and another of $150. For the

first 40 days there is a debit for interest on the \$100 which is represented by the bar-graph (b). A similar bar-graph (a), one and one-half times as wide, pictures the 40 days interest on the \$150. For the remaining 50 days there is an interest debit on the \$150 which is represented by the bar-graph (c).

To obtain Fig. 2, which pictures the bank procedure, a bar-graph (x) which represents the interest on \$100 for 50 days is *debited* and *credited* to the customer's account.

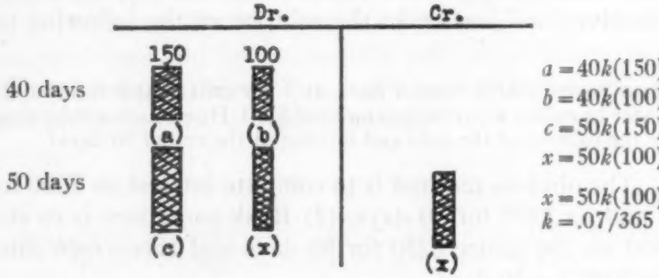

FIG. 2

(1) Expressed in algebraic terms, Fig. 1 indicates that the amount owing at the end of 90 days is,

$$P+I,$$

$$P=250-100=150,$$

$$=\text{original debt}-\text{partial payment}.$$

$$I=(a)+(b)+(c)$$

$$=40k(150)+40k(100)+50k(150)$$

$$=40k(250)+50k(150)$$

$$=40\text{ days int. on }\$250+50\text{ days int. on }\$150.$$

(2) Debiting and crediting $x=50k(100)$ as in Fig. 2.,

$$I=(a)+(b)+(c)+(x)-(x)$$

$$=40k(150)+40k(100)+50k(150)+50k(100)-50k(100)$$

$$=(100+150)(40k+50k)-50k(100)$$

$$=90k(250)-50k(100)$$

$$=90\text{ days interest on }\$250-50\text{ days interest on }\$100.$$

Evidently either of the two methods is applicable to the

general case in which (*n*) partial payments are made to cover (*m*) successive loans. In the banking procedure interest is charged on each loan and credited on each payment.

MODERN PROBLEMS IN WATER SUPPLY AND SEWAGE DISPOSAL*

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Problems in water supply relating to chemical content, hygienic quality and potability are so fundamentally dependent upon the source of the water supply and its exposure or lack of exposure to sewage pollution, that a logical approach would seem to be first a discussion of sewage disposal, followed by a discussion of what can be done to the composition and quality of raw water which has been exposed to sewage contamination, no matter how remote.

Rain, snow and sleet are free from contamination, except for gases and dusts washed out of the ambient atmosphere, but practically all other sources of water supply contain organic plus inorganic solids which have been dissolved out of the rocks and soil or have been added as the residue of human, animal or vegetable wastes.

Sewage residues are the most important solids present in water, from a hygienic standpoint. The inorganic salts present in sewage are largely unchanged, except by dilution, as they pass into surface waters and are in general innocuous unless the concentration exceeds rather liberal limits. On the other hand, the organic solids may be a source of mischief, as they harbor and foment the growth of saprophytic bacteria and may sustain pathogenic bacteria, obnoxious microorganisms and objectionable tastes and odors.

Fortunately sewage treatment stands as a barrier to the entrance of these organic solids and bacterias into water supplies. Polluted water or sewages can be screened, settled, treated with chemicals, freed from suspended solids, oxidized biologically by trickling filters, sand filters or activated sludge, and finally sterilized with chlorine. These processes are primarily physical but biological reactions are the more important, in that they

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transform unstable, putrefiable or odorous organic solids into stable loam-like solids, non-odorous organic residues, or soluble inorganic salts such as carbonates or nitrates.

Sewage Treatment

Sewage treatment may comprise various steps, the degree of treatment depending on the conditions of disposal of the effluent. In some cases, where large dilution is present, as in seacoast cities like Los Angeles, only screening is employed before the sewage is discharged to sea. Screens remove coarse suspended solids and thus improve the appearance of the sewage as it is dispersed in sea water several miles offshore, but the reduction of organic matter by this simple process is very low.

The minimum degree of treatment in most cases is clarification, or continuous-flow sedimentation for one to four hours' holding period in tanks equipped with mechanical sludge-removing rakes, which sweep the bottom of the tanks and collect the solids at one point for removal. Usually about 90 per cent of the settleable solids are removed, or 60 to 75 per cent of the total suspended solids, the remaining 40 per cent being of colloidal nature and non-settleable in the period of time provided. These colloidal plus the dissolved organic solids comprise the largest part of the unstable, putrescible, oxygen demanding solids, consequently we find that only 30 to 40 per cent of the biochemical oxygen demand is removed by sedimentation, as compared with removal of 60 to 75 per cent of the suspended solids.

If the river or receiving body of water is large enough, sedimentation may be all that is necessary, supplemented by chlorination if fairly complete removal of bacteria is desirable.

If conditions of disposal require that more of the colloidal solids be removed, this can be done by addition of chemicals such as alum, lime, ferric chloride, ferric sulfate, or chlorinated copperas, which chemicals flocculate the dispersed solids and increase their rate of settlement so that removals of 80 per cent of the suspended solids and 70 per cent of the oxygen demand can be attained. Use of chemicals is indicated especially when the flow of the receiving stream diminishes greatly in summer and some increased degree of treatment appears necessary.

If diluting water for the effluent is very low, say only one to five volumes, some more effective process of treatment must be

used, and then it is necessary to add biological treatment, usually trickling filters or the activated sludge process.

Trickling filters consist of beds of broken stone 2 to 3 inches in diameter, 3 to 10 ft. deep, with settled sewage distributed over the surface by revolving distributors or sprayed by nozzles placed at about 14 ft. centers. The stone does not act as a mechanical filter but serves as a framework on which slimes of organic matter grow. Within these films are bacteria and larger organisms, of aerobic nature if the filter does not become clogged. These aerobic bacteria transform unstable carbonaceous matter to CO_2 and H_2O, sulfur compounds to sulfates and organic nitrogen to ammonia and later to nitrites and nitrates. Thus if the filter is operating properly the effluent contains most of the organic matter present as stabilized end-products which are non-odorous and unobjectionable. The filter from time to time discharges more or less of the film, which sloughs off, and this solid matter is settled and removed in final settling tanks. The process may remove 90 per cent of the suspended solids and 80 to 90 per cent of the oxygen demand.

Trickling filters are somewhat unsuitable for large cities, as only approximately 3 to 15 million gallons of sewage can be treated per acre per day, representing populations from 20,000 to 100,000, at a normal sewage flow of 150 gallons per capita per day. Trickling filters also may produce odors and frequently are infested with filter flies, *Psychoda alternata*, which may be locally objectionable. However, in spite of these disadvantages, trickling filters are in wide use, especially the so-called "high capacity" filters, which have been developed within the past ten years and operate in the higher range of 15 million gallons per acre per day. These filters were used in many army camps. They have various names—"bio," "aero," "accello," and others.

For large cities the preferred process is the activated sludge process, which was discovered in 1913 in the laboratory of the sewage treatment works in Manchester, England. Prior to this date, many attempts had been made to oxidize settled sewage by blowing air through it, but the time required was so long that it was impracticable. Then Ardern and Lockett discovered, at Manchester, that the time of oxidation could be shortened from three weeks to six hours if, after aeration, the sludge was settled and retained for treatment of succeeding additions of settled sewage. It was found desirable to retain enough sludge

to give a volume, on settling for one hour, of about 20 per cent of the volume of sewage treated.

After it was demonstrated that this process was practical, it required many years to learn how the sludge could be dried, as this type of sludge is very watery and cannot be filtered successfully unless chemicals are added to flocculate the solids. At first sulfuric acid or alum were used as pre-filtration chemicals, but in 1925 a much better coagulant, ferric chloride, was discovered by Palmer at the Calumet Treatment Works of The Sanitary District of Chicago, and this chemical has been used ever since quite widely, as no better coagulant has been discovered in the past twenty years. At first the cost of ferric chloride was six cents per pound, but this has been reduced to one and one-half cents per pound, thus adding to the attractiveness of this chemical.

The activated sludge process removes 90 per cent of the suspended solids and 90 to 95 per cent of the oxygen demand, thus it gives the highest degree of treatment of any process used on a large scale. Settled sewage is aerated in large tanks which have a detention period of from three to six hours. Sludge is settled in final settling tanks, from which it is removed and returned to the incoming sewage. This continues until the volume of sludge is around 20 per cent; thereafter the daily incoming solids are removed and dewatered.

The activated sludge process will handle around 15 to 20 million gallons per acre per day. It is free from odor, and the sludge, after drying, contains about twice as much nitrogen as any other sludge, and therefore is worth enough as a fertilizer to warrant a rather expensive process of dewatering. The dried sludge contains about 6 per cent nitrogen and 3 per cent phosphoric acid and sells for $10 to $15 per ton when shipped to the Southwest U. S., where it is used for fertilizing tobacco and cotton. The sludge is an excellent fertilizer for grass and foliage, but contains practically no potash, hence must be fortified with potash and phosphoric acid to make a complete fertilizer.

The Sanitary District of Chicago has the largest activated sludge plant in the world, the Southwest Treatment Works, also the next largest, the North Side Treatment Works, and a smaller plant, the Calumet Treatment Works. All of these plants treat the sewage from tributary areas of the District to the highest degree practicable. One more plant, the West Side Treatment Works, is only a primary clarification plant,

but this will soon be augmented by activated sludge treatment.

There are many problems in operating an activated sludge plant. As the process is aerobic, enough air must be supplied to produce 2 to 5 parts per million dissolved oxygen at the end of the aeration tanks. Too much air is wasteful, and too little allows the bacterial flora to become anaerobic and ineffective. The air must be diffused through porous silica or alumina plates to produce fine bubbles, but on the other hand the pressure loss in passing through the plates must be kept very low, usually less than one pound (plus the pressure to overcome the water depth of 15 feet in the aeration tanks, equivalent to 6.5 pounds). In warm weather we have had a heavy growth on the diffuser plates of a water mold of the *saprolegniales*, which increases the air pressure rapidly. We have treated the diffuser plates with many fungicides, and in the laboratory have found pyridyl mercuric stearate to be best, but the conditions of operation, with constant leaching and aeration of the plates, make a rather difficult problem to apply a fungicide that will be effective for several months, and this problem has not yet been solved.

Other problems in sewage treatment include: reduction of cost of dewatering activated sludge; use of sewage sludge as fertilizer; longevity of pathogenic bacteria or viruses in sewage and effluents; effects of industrial wastes; and sewage treatment in military installations. The last is under study by a subcommittee of the National Research Council.

Discharge of effluents into streams is the common method of disposal. Effluents from the treatment works of the Sanitary District are discharged to the North Shore Channel, Main Channel or Calumet Sag Channel, thence to the Des Plaines River at Lockport, and eventually to the Illinois River. For many years studies have been made of the sanitary condition of this system of waterways, with laboratories and chemists at Joliet, Marseilles and Peoria. There is a gradual improvement downstream due to processes of self-purification, which include dilution, aeration, sedimentation, antibiosis and photosynthesis. The dilution ratio is very low, less than one to one in the upper river, because of the U. S. Supreme Court limit of 1,500 cu. ft. per second diversion of lake water. This very low dilution makes it necessary to treat Chicago's sewage to the highest practicable degree, consequently the sewage treatment program for the Sanitary District has been and will be the largest and most costly in the world.

Water Supply

Although the Illinois River is not used for water supply throughout its course, other large streams such as the Mississippi and Ohio are used for many sources of raw water and the processes of water purification must be able to take this polluted water and to produce a safe, attractive potable supply. This necessity also applies to cities in the Calumet Region of Indiana, where pollution of Lake Michigan by industrial wastes makes it difficult to remove tastes and odors as well as all danger from bacterial contamination.

Rapid sand filtration is used on most surface waters, with chlorine applied in various ways, for producing a safe potable supply. The water is first treated with alum, ferric salts, or sodium silicate, then flocculated in slow-mix basins to build up a readily settleable floc, which is settled in basins of several hours detention period. The supernatant water is passed through rapid sand filters, which are backwashed to remove the floc from the surface of the sand. Chlorine is applied, usually prior to sedimentation, plus a smaller dose for final disinfection.

Bacterial standards set up by the U. S. Public Health Service set a limit of 50 coliform organisms per 100 cu. cm. of raw water as the maximum that should be treated by chlorine alone, whereas with flocculation, sedimentation, filtration plus chlorination, the limit can be 5,000 or more coliform organisms per 100 c.c. (the "coliform index"). Thus it is apparent that filtration is very effective, as compared with chlorination alone.

Chicago's new "South District Filtration Plant" is the largest water filtration plant in the world, with an average capacity of 320 million gallons per day. Flocculation, sedimentation and chlorination are now in use and filters are hoped to be completed next spring.

Filter alum is usually used for flocculation. Normally, if tastes and odors are not present in the raw water, alum can be used alone, followed by pre-chlorination, filtration and post-chlorination. When it is necessary to control tastes and odors, sometimes ammonia is added to produce chloramines which do not give as pronounced a taste with certain industrial wastes as free chlorine. Usually activated carbon is used to reduce tastes and odors and this is applied prior to sedimentation, with complete removal by filtration.

Modern developments for improving filtration and taste removal include the use of sodium silicate, properly treated

with sulfuric acid, for improving flocculation; "breakpoint" chlorination or chlorine dioxide for removing organic matter and taste-producing substances; aeration for improving taste; and synthetic resins for removing dissolved salts. Most of these refinements are costly and not yet applicable to large pumpages, but are in use on smaller installations.

Thus new discoveries are extending the field of recovery of potable water from contaminated sources. The limit of pollution is set more by aesthetic than scientific standards, unless the source of supply is so critical that an objectionable raw water must be used. Then the various chemical correctives must be used, sometimes at considerable expense.

The sewage-water cycle is still under technical control if funds are provided to reduce pollution at its source by means of sewage treatment works. The deteriorated condition of many of our streams indicates that a national program of sewage treatment is urgently needed.

THE QUIZ SECTION

Julius Sumner Miller

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1. It is a fact that the earth is closer to the sun in January than in July. (T or F)
2. If the earth rotated fast enough so that the centrifugal force on a body at the equator were equal to the body's weight, how long would the day be?
3. All living matter has carbon in it. (T or F)
4. What is meant by biotic potential?
5. Pair the following: citric, tartaric, malic, lactic; lemons, grapes, apples, milk.
6. A ring and a solid disc of the same mass and radius roll down an inclined plane. Which one wins?
7. Tall chimneys "draw" better than low ones. (T or F)
8. The apparent diameter of the sun is practically the same as the apparent diameter of the moon. (T or F)
9. A force of two pounds is required to stretch a spring one inch. How much work must be done in stretching it six inches?
10. The velocity of water in a river is greater in mid-stream than it is near the bank. (T or F)

ANSWERS TO THE QUIZ SECTION

1. T; 2. One hour, twenty-five minutes; 3. T; 4. Rate of reproduction; 5. Citric-lemons, tartaric-grapes, malic-apples, lactic-milk; 6. The solid disc; 7. T; 8. T; 9. 3 ft-lbs; 10. T.

DERIVING MOMENT OF INERTIA FORMULAS WITHOUT CALCULUS

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The first formulas to be derived here are for the moments of inertia of a bar whose mass is distributed uniformly along its length L, about an axis of revolution through (*a*) one end of the bar, (*b*) its center, and (*c*) at any point at a distance d lengthwise from its center. Next to be derived are formulas for the moments of inertia about an axis at either end of a bar whose mass increases uniformly from one end to the other.

Suppose a bar of uniform mass be divided into n equal parts, each part of mass M/n and length L/n. Then the moment of inertia of the bar about an axis at one end is, approximately,

$$I_E=\frac{M}{n}\left(\frac{1}{2}\,\frac{L}{n}\right)^2+\frac{M}{n}\left(\frac{3}{2}\,\frac{L}{n}\right)^2+\cdots+\frac{M}{n}\left[\frac{2n-1}{2}\,\frac{L}{n}\right]^2$$

$$=\frac{M}{n}\,\frac{1}{4}\,\frac{L^2}{n^2}\,[1^2+3^2+5^2+\cdots+(2n-1)^2].$$

One way to sum the series in brackets is:

$$1^2+2^2+\cdots+(2n)^2-[2^2+4^2+\cdots+(2n)^2]$$

$$=\frac{2n(2n+1)(4n+1)}{6}-2^2(1^2+2^2+\cdots+n^2)$$

$$=\frac{2n(2n+1)(4n+1)}{6}-2^2\cdot\frac{n(n+1)(2n+1)}{6}$$

$$=\frac{(4n^2-2n)(2n+1)}{6}\,.$$

Hence

$$I_E=\frac{ML^2}{4n^3}\,\frac{(4n^2-2n)(2n+1)}{6}=\frac{ML^2}{24}\cdot\frac{8n^3-2n}{n^3}$$

$$=\frac{ML^2}{24}\left(8-\frac{2}{n^2}\right).$$

As n becomes infinite,

$$I_E=\frac{ML^2}{24}\,(8)=\frac{ML^2}{3}\,. \qquad (1)$$

To derive the moment of inertia of the bar about an axis through its center, add the moments of inertia of the two halves about an axis through their ends, from Eq. (1). Thus,

$$I_C=\frac{\frac{M}{2}\left(\frac{L}{2}\right)^2}{3}+\frac{\frac{M}{2}\left(\frac{L}{2}\right)^2}{3}=\frac{ML^2}{24}+\frac{ML^2}{24}$$

$$I_C=\frac{ML^2}{12}\cdot \tag{2}$$

Similarly for the moment of inertia of the bar about an axis through a point at a distance d lengthwise from its center, we have,

$$I_A=\frac{\frac{M\left(\frac{L}{2}-d\right)}{L}\left(\frac{L}{2}-d\right)^2}{3}+\frac{\frac{M\left(\frac{L}{2}+d\right)}{L}\left(\frac{L}{2}+d\right)^2}{3}$$

$$=\frac{M}{24L}\left[(L-2d)^3+(L+2d)^3\right]$$

$$=\frac{M}{24L}\left(2L^3+24Ld^2\right)$$

$$I_A=\frac{ML^2}{12}+Md^2. \tag{3}$$

If the mass of the bar increases uniformly from left to right, the moment of inertia about an axis through the left-hand end is found thus. Divide the bar into n sections, each having a length L/n. The magnitudes of the masses form an arithmetic progression from left to right, with the first term equal to the common difference and the sum equal to M. Hence, $M=\frac{1}{2}n(na)$, where a = first term = common difference. Then $a=2M/n^2$. The rth term, $r<n$, is $2rM/n^2$. This makes the moment of inertia I_L about an axis through the left-hand end equal approximately to

$$\frac{2\cdot 1\cdot M}{n^2}\left(\frac{L}{n}\right)^2+\frac{2\cdot 2\cdot M}{n^2}\left(\frac{2L}{n}\right)^2+\cdots+\frac{2\cdot n\cdot M}{n^2}\left(\frac{nL}{n}\right)^2$$

$$=\frac{2ML^2}{n^4}\left(1^3+2^3+\cdots+n^3\right)=\frac{2ML^2}{n^4}\cdot\frac{n^2(n+1)^2}{4}$$

$$=\frac{ML^2}{2}\left(1+\frac{1}{n}\right)^2.$$

As n increases without bound,

$$I_L=\frac{ML^2}{2}\cdot \tag{4}$$

The moment of inertia I_R about an axis of rotation through the right-hand end is, approximately,

$$\frac{2(n-1)M}{n^2}\left(\frac{L}{n}\right)^2+\frac{2(n-2)M}{n^2}\left(\frac{2L}{n}\right)^2+\cdots$$

$$+\frac{2(n-n)M}{n^2}\left(\frac{nL}{n}\right)^2$$

$$=\frac{2nM}{n^2}\left(\frac{L}{n}\right)^2+\frac{2nM}{n^2}\left(\frac{2L}{n}\right)^2+\cdots+\frac{2nM}{n^2}\left(\frac{nL}{n}\right)^2$$

$$-\left[\frac{2M}{n^2}\left(\frac{L}{n}\right)^2+\frac{2\cdot 2M}{n^2}\left(\frac{2L}{n}\right)^2+\cdots+\frac{2nM}{n^2}\left(\frac{nL}{n}\right)^2\right]$$

$$=\frac{2ML^2}{n^3}\,(1^2+2^2+\cdots+n^2)-\frac{2ML^2}{n^4}\,(1^3+2^3+\cdots+n^3)$$

$$=\frac{2ML^2}{n^3}\,\frac{n(2n^2+3n+1)}{6}-\frac{2ML^2}{n^4}\,\frac{n^2(n+1)^2}{4}$$

$$=\frac{ML^2}{3}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)-\frac{ML^2}{2}\left(1+\frac{1}{n}\right)^2.$$

As n increases without bound,

$$I_R=\frac{2ML^2}{3}-\frac{ML^2}{2}=\frac{ML^2}{6}\cdot \tag{5}$$

It is of course better to derive the above formulas (1) to (5) by calculus if the students have had this subject. If they have not, the proofs given here are satisfactory.

Calcimine is basically a physical structure held together with glue. The glue not only holds the powdery material together but will fasten a film of the material to a plaster surface. Animal hide glue is used in hot water calcimine and bone glue in cold water materials.

NEW GEOGRAPHIC CONCEPTS OF MAN'S PLACE IN THE WORLD*

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Revolutionary changes in transportation always produce revolutionary changes in human geography. The introduction of the steamship and the railroad a century ago increased the speed of travel so much that distant continents became more accessible than neighboring states had been a few decades earlier. This acceleration in transportation made possible the growth of industrial centers based on the importation of raw materials from far-off countries and the exportation of finished products to markets throughout the world. The rise of the industrialized state in turn, led to commercial competition and a mad scramble for colonies and markets in the less civilized portions of the globe, especially after 1870. The final fruit of this international scramble was at least one world conflict, if not two.

Recent technological developments in the field of transportation and communication have already effected innumerable changes in all aspects of human geography. Owing to the development of the airplane in recent years the world has finally become a community of nations in which individual countries can no longer plan solely in terms of their own welfare. Most people in the United States and in many other countries do not yet recognize this fact. In our country, the all too rapid return to prewar antagonistic attitudes toward other peoples and places offers evidence of our lack of worldmindedness. This is unfortunate. The development of aerial transportation may very well result in yet another world struggle if steps are not now taken to prevent the growth of fear and suspicion among great powers. Whether we like it or not, the airplane and the destructive bombs which it can carry have reshaped the earth as a home for man. They have also outmoded many traditional man-land relationships that have existed for centuries.

These changes in human geography must be studied and analyzed if we are to benefit from the development of the airplane and atomic energy. Not to do so will mean that we can plan only in terms of survival rather than progress. If we want to

* Presented to the Geography Section of the Central Association of Science and Mathematics Teachers, November 23, 1945.

move forward toward world peace, we must (1) define clearly the impact of the airplane on our traditional spatial and ecological relationships, (2) comprehend the nature and variety of the unresolved geographic problems resulting from the development of the airplane and nuclear fission, and (3) provide a place in our educational system for teaching up-to-date understandings of geographic relationships and for training in the techniques of thinking geographically.

Before discussing these at greater length, I should like to point out certain geographic axioms:

(1) All men and regions are spatially interrelated. The geographer treats the world as a whole, as one object of study. He considers localities and men as only parts of this whole.

(2) Man is related to the elements of his natural environment. These are his ecological relationships, and what man does is conditioned in part by the natural make-up of the area in which he lives and, in part, by his inherited cultural traditions and technologies.

(3) Spatial and ecological relations of man differ from place to place.

(4) Man's spatial and ecological relationships are constantly changing.

These axioms must function dynamically in the kind of geographic thinking which should prevail not only for the immediate post-war period, but for a much longer time.

Impact of the Airplane on Spatial Relationships

The developments in air transportation since 1938 have made the average person conscious that the world is spherical. This fact now permeates and illuminates our thinking about man's spatial relationships in 1945. Not only do we now know, but we are at last interested in the fact that the shortest distance between two points on the earth's surface is an arc of the great circle which passes through thse two points. Our B-29's, on a non-stop flight from Hokkaido to Washington, D. C., on November 1st, followed such a route. From the northernmost Japanese home island, the bombers flew to Washington by way of Agattu, Kodiak, Sitka, Fort St. John, Winnipeg, and Detroit—a distance of only 6544 miles.

This "new" geographic concept has awakened the public's interest in maps. Cartographers are having a field day creating, or rather re-creating, fantastically-shaped reproductions of the

world. They have invented a variety of projections to show our new spatial relationships. Some mapmakers even predict that the polar projection will become the general map of the new age because in all azimuthal projections centered on the capitals of leading nations, the north pole lies close to the center of the map.

Maps are also being used effectively by a variety of organizations. Chambers of Commerce have discovered that maps can be made which show Kokomo, or Kalamazoo, or Honolulu occupying a vitally important, central position in the world. Advertisers and propaganda agencies have found maps useful as a medium for reaching the American citizen, in part because he is so illiterate in the field of map interpretation.

These uses, and misuses, of maps demonstrate two things: (1) that there is need for instruction in the proper selection and use of map projections, and (2) that our high school and college graduates need more specific training in map reading.

The use of great circle routes has shortened *travel distances* between two places. *Time distances*, too, have been shortened in recent years. We, in the United States, are now no more than forty-eight hours away from any other people in the world. In October of this year, Army Transport Command planes encircled the world in less than 150 hours. Whether we like it or not, we are now next-door neighbors of every other member of the community of nations.

These developments in transportation have affected the relative location of all places in the world. No longer can we classify a settlement as being accessible, nearly accessible, remote, or isolated. As long as suitable run-ways exist on which to land, to refuel, or to make repairs, all places are now accessible. Isolated spots no longer exist, and the theory of isolationism is wholly anachronistic in an air-age. This fact cannot be overstressed. It needs to be brought to the attention of the American people again and again for in recent months we have paid lip-service to the general principle of international cooperation without doing much to further world unity.

During the war many guesses were made, some unfortunately by geographers, about the location of postwar airways. It was claimed that the Arctic area would become the future Mediterranean region of the air age. "The importance of the Arctic in an air age," it was argued, "depended upon the great circle routes which crossed the north polar area from one place to

another." It was pointed out, for example, that the shortest distance from Panama to Singapore was by way of the Arctic, and that the great circle route from New York and the north-central industrial heart of the United States to Chungking was dead across the pole, a distance of 6500 miles.

Since the end of the war, American-controlled air lines have charted trans-oceanic routes to foreign countries. None of these is scheduled to go by way of the Arctic Ocean. On August 27, CAB examiners recommended that Northwest Airlines, Inc., and Pan-American Airways be licensed to fly Pacific routes to China, Japan, and India. They advised that Northwest be authorized to fly between the co-terminal points of New York and Chicago and Manila, by way of Edmonton, Anchorage, Paramashiro, the Kurile Islands, Tokyo, Shanghai, and Hong Kong. The examiners recommended further that Pan-American's Pacific certificate be amended to extend its Central Pacific route from Midway Island to Calcutta, via Tokyo, Shanghai, Hong Kong, and Bangkok. This route is far from the conjectured "shortest" distance between North American and Asiatic centers.

Across the Atlantic, the air routes approach great circle routes, but do not follow them. Here, too, the airways already charted indicate that there will not be much traffic by way of the north polar region.

These postwar air plans suggest that there is a difference in the location of routes for military strategy and their location for commercial purposes. In times of war, speed is all important. Airplanes then fly shortest distances, or great circle routes, from one place to another. In times of peace, commercially controlled airplanes fly where trade flourishes. Because it does not pay for commercial lines to go via the Arctic Ocean, other routes, longer in distance, have been selected. Prewar airlines then are being extended rather than re-located.

This does not mean that polar routes should be abandoned. Certain air routes must be maintained for military reasons. It is possible that in the future we may be attacked, not by way of Pearl Harbor, but by way of the Arctic Ocean. Our only possible defense against such action may be the establishment and maintenance of airports northward from the interior of the United States. This fact resolves the debate over the Alaskan highway. Canada willing, this road will become a necessary adjunct to

the airports which must be built to protect our most vulnerable front.

Finally, our horizons have been enlarged by the development of air transportation. Since 1938, we have established new spatial relationships with unfamiliar places throughout the world. The islands of the Pacific, in particular, have at last become well-known. These islands which have played such an important role in our war strategy are now destined to function even more prominently in our plans for world peace. Our congressional representatives recommend that for our own security, the United States should have at least administrative control over the former Japanese Mandated Islands of the Marshalls, the Carolines, the Mariannas, and the outlying Japanese islands of the Izus, Bonins, and Ryukyus. It is certain, at least, that the United States must not permit these Pacific bases to lapse into a state of unpreparedness as we did at Guam and Wake. Full commercial utilization should also be made of the islands retained. Whether we get full title to these lands, or control them under some kind of world government, they have, during this air age, become strategically important to us.

Impact of the Airplane on Man's Ecological Relationships

Growing out of the four spatial relationships discussed above are several ecological changes that need to be examined in light of the recent developments in air transportation. If the airplane does anything it discounts natural barriers, particularly the barrier of distance. This is one of the reasons why traveling by air has become so popular. On journeys which take only a few hours by train, there is no great incentive to go by air unless one is in a hurry. When long distances must be covered, however, the airplane affords great advantages. In addition, air travel offers a means of crossing certain kinds of landforms and vegetation zones which have in the past acted as barriers to transportation. Much of the physical make-up of South American countries is such that there are extensive areas where only the airplane can provide rapid transport. Trips, like that from Barranquilla to Bogota which formerly required days can now be made in relatively few hours by plane. Likewise, regular air routes have become well established in Peru where mountain communities are being serviced regularly by scheduled air flights. Moun-

tains and tropical rain forests can now be discounted as barriers to the movement of men and goods.

Another effect of the air age has been a growing interest in the wind systems of the world. In the future we shall become more and more conscious of the speed, direction, and quality of wind. Airplanes rarely fly in still air. Instead, planes encounter a down-wind, in which case their speed is increased, or an up-wind. In this case, the plane's traveling time is substantially increased. Because of headwinds, Lieutenant-General Jimmie Doolittle recently failed to set a non-stop transcontinental speed record from Oakland, California to Washington, D. C. Asked why his ship did not set a new record, Doolittle replied, "We didn't get the predicted winds." Near Salt Lake City his plane hit a seventy-mile wind. As soon as this happened, Doolittle realized that a new record would be impossible.

It is now conceivable that future air routes may be planned not in terms of great circle routes, but in terms of the character of the prevailing winds. CAB, for example, has already amended the foreign air carrier permit of the British Overseas Airways Corporation to allow that airline to conduct operations through the winter months from Great Britain and Northern Ireland to Baltimore, Maryland, via Lisbon, West Africa, Trinidad, and Bermuda. This allows the British line to fly the South Atlantic route during periods when operations are not practical over the North Atlantic route because of wind conditions.

Still another ecological change is the emergence of inland centers as terminal ports. Transportation by air makes it unnecessary to break cargoes at shore lines. Passenger and freight flights can now be terminated at any inland point served by an airline. Thus, Pan-American which in the past flew only from seaboard cities has been grated permission to fly inland to the major traffic centers of this country. This development has led many Midwestern cities to think in terms of becoming world air hubs. Enterprising citizens of Chicago envision their city as the American terminus of the new Pacific sky route to the Orient. Representatives from Seattle and other West Coast cities are busy protesting any such development. They claim that the relocation of Pacific trade from its "natural sources in the western states" to the already over-concentrated industrial Midwest would be an artificial development. Seattle, which in the past has led in trans-Pacific trade, may find itself left without direct air connections with the Orient if the advice of CAB

examiners is followed. Under the recommendations made by these men, Seattle passengers and express would have to travel north by a feeder line to Anchorage, or south to San Francisco. From an industrial point of view, this is not detrimental to the nation as a whole. Records show that in 1940, out of 295 companies in the United States having investments in the Orient, 166 or 56% were located in New York, and 218 or 74% in the northeastern or middle eastern states. A route originating in the East or Midwest would certainly be more convenient for businessmen than one originating in Seattle. On the other hand, the bulk of passenger traffic between the United States and Asia has always been furnished by West Coast centers.

Finally, aerial photographs have become unexcelled as base maps for many kinds of geographic data. Such pictures taken for mapping purposes have at least two advantages over ground surveys. An aerial survey can be made in shorter time, and areas that are inaccessible or difficult to map from the ground impose no special problem to aerial surveyors. During the war, aerial photography proved most useful. In a single morning over 4500 square miles on the final Tunisian front were mapped by plane. These photographs made it possible, according to Colonel James G. Hall of the Army Air Force, for our artillery to fire so accurately that it scored direct hits on enemy guns in many instances, and in no case was a position missed by more than five yards.

In addition to the development of aerial photography as an aid to the propagation of war, aerial photographs have many peacetime uses. They serve the geographer exceptionally well in the study of patterns of human occupance. The existence, distribution, and characterization of settlement forms, land types, routes of access, and other landscape features are usually revealed by aerial photographs better than by studies on the ground.

Radar, too, will further geographic research. Owing to its development, the precise visual identification of objects from the air is no longer hampered by haze, clouds, or darkness. No country today can keep secrets that may be detected from the air. Photographic planes are now prepared for both daylight and night reconnaissance. This fact alone makes it necessary to think in terms of the geographic changes that have been brought about by the advent of the air age. If in the years ahead we keep such facts as these in the foreground of our thinking, we can be more hopeful that world peace is attainable.

Unresolved Problems of the Air Age

The air age has produced several problems which deserve special attention at the moment. Heading this list is the need for the establishment of geo-political unity in a world where geophysical unity already exists. We have conquered the problem of physical power; we have harnessed natural forces so that we can destroy the earth and all that man has created; we have mastered the mechanics of travel on land, on sea, and in the air; and we have solved the problem of transmitting sound and image through space. These developments have freed most people from the limitations imposed by their immediate surroundings. In turn, they have made peoples more and more dependent on foreign regions for their security as well as for their livelihood. National self-sufficiency—economic, political, and social—is as much out-of-date in an air age as the horse and buggy. It is time that this fact be recognized. We must develop a world consciousness now. The first step to this end is the development of world-wide political unity.

A second important problem is related to the disappearance of natural barriers. To Americans who have long depended upon the broad extent of the Atlantic and Pacific Oceans for protection from invasion this is indeed a serious problem. Topography forms a defensive barrier only when weapons are relatively weak. In an age when rockets can streak across hundreds of miles to factory targets, when jet-propelled bombers can plow through the substratosphere at 1400 miles an hour carrying 50-ton bombs, when aircraft have sufficient range to attack any spot on the earth and return to a home base, the air approaches to a country, and not its sea or land approaches, become the principal points of importance. General Arnold only recently warned us of this. Gone are the days when a natural barrier can be used as a means of national defense.

Finally, there are the many problems growing out of the location of airports within a city. Air transportation, unfortunately, followed, rather than preceded the development of industrial and commercial centers. Large cities are so solidly built up that airports must of necessity be constructed on their outskirts. This gives small communities an advantage over big cities as sites for the development of new industries which will ship their goods by air. It is probable that the airplane may become an influence in the future decentralization of industry.

More Training in Geography

How can these problems be solved? The world is changing continually, and man must learn how to adjust himself to his ever-changing environment. What can we do to learn how to adapt ourselves to, and how to control, the natural forces which the mind of man can set loose? One thing must be clear to all of us. At the bottom of some of the reasons why the world is not a peaceful, contented place in which to live lie certain geographic illusions—the illusion of self-sufficiency, the illusion of cultural and natural conservatism, the illusion of power politics, the illusion of isolationism. We must prepare ourselves and our students for the air age, the age of atomic bombs, radar, and other technical developments. To do this we must teach them that every nation and every group of people differ from place to place. Next we must teach that these differences must be understood, and in many instances tolerated and respected. Finally, we must learn that all places and all peoples are interdependent, and that the traits and aspirations of each nation affect all others. We must set out, here in America, to educate ourselves and the coming generation to know the world and the people who live in it. If we want to live in a peaceful world, much depends upon how well we master this task. We must teach our students to incorporate new geographic concepts into their thinking about world affairs, for upon these concepts will depend their attitude toward their fellow inhabitants of this earth and all of their concepts of equality, justice, freedom, and security. These are questions for another day. For the present let's begin by knowing our ever new, ever smaller, ever crowding house in which we must of necessity all live together in harmony if we are to live at all.

DDT SAVES LARGEST TREE IN PACIFIC NORTHWEST FROM INSECT ATTACK

DDT recently saved a giant fir in Clatsop County, Oregon, believed to be the biggest tree in the Pacific Northwest, from destruction by loopers, which are swarming caterpillars that constitute one of the worst of timber pests. The forest giant has a diameter of more than 15 feet and is claimed to be more than a thousand years old.

Rescue of the huge fir was an incident in a general campaign to stop the ravages of the looper in Oregon softwood forests. DDT seems to have scored an outstanding success in this fight. Counts of dead loopers ran as high as 480 on six square feet of ground beneath the trees.

IMPROVEMENT OF LABORATORY TECHNIQUES THROUGH MOTION STUDY

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Marygrove College, Detroit, Michigan

The worker's comfort is a factor which is badly neglected in school as well as in many commercial laboratories. It is well known that precision, output, and efficiency depend upon the ease with which the operator performs his tasks. An investigation of the movements of the laboratory worker by using the technique of time and motion study can illuminate inefficient methods and serve as a first step toward improvement.

In the study presented below, two frequently used operations were checked: the weighing and the transfer of precipitates. Both operations as usually carried out necessitate an awkward, bent body position, since the operator must be in a position to observe the pointer of the balance or to bend down to the wash bottle.

The technique of studying these operations consisted in photographically recording these movements by strapping electric bulbs (6V) to the head and right wrist of the operator. The investigation was carried out in a room supplied with only enough light to permit ease of operation. In this way, the bulbs on the head and wrist produced imprints on a photographic plate, but other objects in the room were not recorded. The most typical position assumed during the operation was photographed on the plate by a flashbulb shot.

Plate I shows the posture assumed by the operator to read and record the movements of the pointer when the balance is in standard position on a 36 inch table. The tiring, bent position of the head and body and the angular movements necessary to place the weights on the pans are highlighted.

The standard balance position fails to provide convenient space for the weights as well as for the notebook. They have to be placed to one side of the balance and only be used by reaching outside the proper area of action. Any vibrations caused by the operator's arms are imparted to the table, thus disturbing the balance.

Plate II shows an improved position of the balance. It was placed on a wall bracket four inches above the table surface. This relieved the strained posture of the operator. Unnecessary motions of the head were eliminated as can be seen by compar-

Photography by Elaine Cousino, Marygrove College

PLATE I. Weighing operation: Balance placed on 36 inch table.

PLATE II. Weighing operation: Balance placed on wall bracket 4 inches above a 36 inch table.

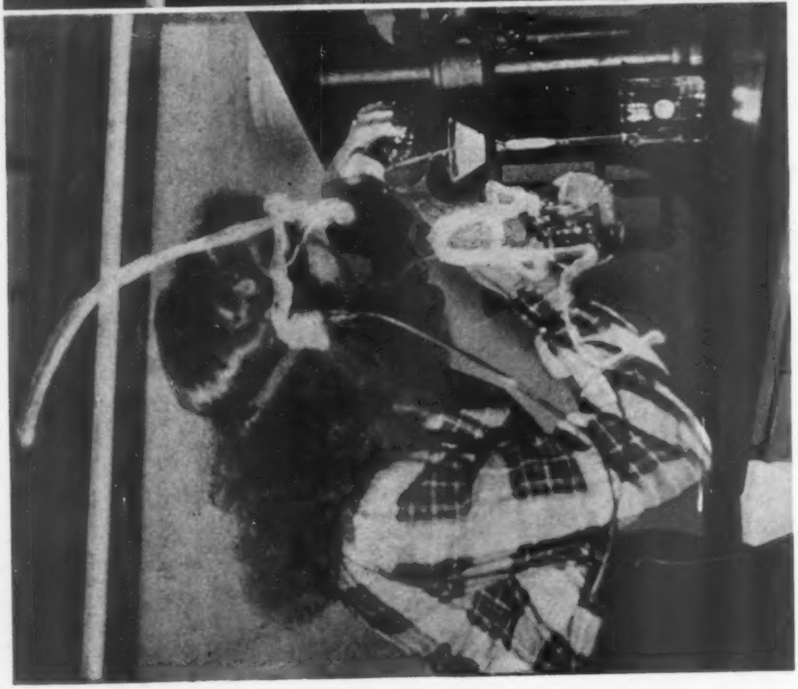

PLATE III. Transfer of precipitates by use of a standard wash bottle.

PLATE IV. Transfer of precipitates by use of a mouthpiece and a five inch rubber tubing on the wash bottle.

Photography by Elaine Cousino, Marygrove College

ing the white streaks caused by the "head lights" in Plates I and II. Further comparison of the two plates shows that the improved position eliminates unnecessary cramping and tension in the movements of the right hand.

In position II, the weight box can be placed flush with the edge of the balance, because its lid will slide under the wall bracket. This arrangement also provides ample space for the notebook so that it may be moved freely on the table without disturbing the balance. A further convenience would be the use of a chain or keyboard balance to eliminate the long right hand motion required for moving the rider.

It is not certain that it is time saving to place the balance on a wall bracket. The advantage lies in the fact that by bringing the balance into a vibration free position, and by decreasing the strain on the operator, the efficiency is improved.

Plate III illustrates the standard method for transfer of precipitates. The worker must assume an awkward head and body position to get the nozzle of the wash bottle in position for transfer. Breathing is hard and the worker becomes tired quickly.

Plate IV shows how transfer of precipitates is made more convenient by attaching a five inch long rubber tubing to the wash bottle. This permits the worker to stand erect during the transfer. Fatigue of the muscles is eliminated; breathing is no longer hard; and the worker has greater endurance. The same improvement could be achieved by use of wash bottles with syringe bulbs.

The transfer of precipitates by the use of a five inch rubber tubing attached to the wash bottle not only adds to the comfort of the operator, but also considerably shortens the time required for the transfer.

Summary

A motion study of the weighing operation and the transfer of precipitates is here presented. The advantages of placing the balance on a wall bracket are pointed out. The use of a rubber tubing with mouthpiece or a syringe pressure bulb on the wash bottle is recommended for the transfer of precipitates. The use of other time and motion studies in the laboratory is advocated to increase the efficiency and the comfort of the operator.

WHAT IS PROBLEM SOLVING IN ARITHMETIC?

J. T. JOHNSON, *Chairman*
Chicago Teachers College, Chicago, Illinois

Considerable interest is being shown right now in the verbal problem in our school textbooks. The reading requirement is being investigated in a city-wide experiment in Chicago, comprising not only the elementary schools but the high schools as well. Perhaps all we can say with certainty now is that reading is necessary but not sufficient.

Four factors which are known as contributors to problem solving were selected as the basis for discussion in this panel. How much each factor contributes will be brought out by the members of the panel.

This discussion will not settle the question of problem solving. One educator says that the purpose of arithmetic is not problem solving. Another question that arises in our minds is, Is the textbook problem the kind of problem that the child will meet in later life?

It is hoped that this discussion will clarify the part played by each of the four factors, i.e., where one factor ends and the other begins, so that we can more intelligently know how to proceed in a program of improvement in the several factors.

Perhaps the need for better and more real and life-like problems in textbooks will be seen as a result. That alone will be worth while.

The solution of the problem of problem solving is a major trend right now in the teaching of arithmetic. Let us hope that the end of this decade will see a part of this solution.

IS IT READING?

Elizabeth F. Jeffords, Acting Principal, Motley School, Chicago

This question cannot be answered by a final and definite "Yes, it is reading." Reading does, however, play an important part in solving arithmetic problems; reading is necessary but not sufficient. Before any attempt to solve an arithmetic problem can be made, there must be an accurate interpretation of all the words and symbols used to express the ideas of the problem. For a successful interpretation, the pupil frequently must draw upon his reading ability. Skill in manipulating numbers is more

easily perfected than is the ability to solve problems. The cause of this has frequently been attributed to the pupils' inability to read their problems with understanding.

Reading is prerequisite to thought; it is the spark which ignites the thinking necessary for solving the problems. It is important, therefore, to make pupils aware of the nature of reading in the mathematics field. Skimming has a place in some reading fields but not in that of arithmetic.

Careful reading of problems should be encouraged. Many times there is a tendency, in order to expedite matters, for the teacher to read aloud textbook problems, explanations or to give oral directions. Instead of this procedure it might be helpful to direct the class to read silently such problems, explanations and directions. Follow the reading by a class discussion or by asking pupils to carry out the printed directions at the blackboard. An arithmetic reading guidance lesson preceding the formal arithmetic instruction is well worth the time devoted to it.

One of the most important reading factors in finding the solution of arithmetic problems is that of the specialized vocabulary. Just as it is necessary to develop meanings of nouns and verbs in basic reading so is it necessary to teach, with orderly and continuous development, the words peculiar to the field of arithmetic. Arithmetic has many such words as well as numerous symbols. A well planned developmental reading program, through the grades, will familiarize the pupils with these words and with the forms of expressions common to arithmetic problems. This familiarity is necessary because in order to think competently in any field the pupil should have a stock of words from which to draw.

The ordinary symbols for addition, subtraction, multiplication and division are learned early in the grades as are the words add, subtract, multiply and divide. Along with these, however, must be developed the meaning of many words which have a special arithmetical meaning as well as a general meaning. For example, valid concepts, in arithmetic settings, must be built for such words as net, yard, principal and countless others. Reading can do much to clarify this difficulty.

Because arithmetic is a system of ideas it exists and grows only in the mind of the learner. It must be learned in understood sequences or relations. Seeing relationships is still another reading skill which will be a definite aid in solving problems. The

necessity of seeing relationships need not be confined to number relations. Pupils require practice in forming mental pictures of what a problem tells them so that they may see the relationships of the various parts to the whole. This requires thinking methods. Reading may be used as a method to discover what the reader knows about the problem and what he doesn't know.

The general ability of reading to predict an outcome or form a judgment is related to the specific arithmetic ability of reading to estimate an answer. Making estimates is an excellent habit to form because it frequently prevents giving an unreasonable answer.

Teachers have sometimes thought that it was the size or variety of numbers given in problems which confused the children. Experiments in many grades have shown that children have greater difficulty telling how to solve a problem without numbers than one with numbers. Practice, however, in reading and discussing problems without numbers, helps the pupil to visualize the situation and to focus his attention solely one meaning.

In conclusion I feel that it is safe to say that a pupil needs to draw upon many of his reading skills in order to solve arithmetic problems. He must be able to understand the specialized vocabulary of arithmetic problems, to distinguish between relevant and irrelevant statements, to interpret directions and explanations correctly, to read with understanding accompanying graphs and diagrams, to recognize relationships between ideas in problems and his own experiences, to form estimates of the answer and to clearly visualize the stated situation.

Reading is not the complete answer to difficulties in solving arithmetic problems but experience has shown me that teachers who make constant use of reading guidance in arithmetic, utilize one of the most valuable sources for pupil development through reading.

Is It Number Relations?

G. T. Buswell, University of Chicago, Chicago, Illinois

If I were dealing with this question as a whole, namely, What is problem solving in arithmetic, I should want to include all four of the subdivisions which have been assigned to the individual members of this panel. For example, an ability to read effectively is essential for understanding a problem in arithmetic and for the initial attack upon it. However, in limiting my part

of the discussion to number relations I should like to point out three ways in which an understanding of number relations is essential to problem solving.

In the first place, the estimation of an answer involves a type of comparison and judgment which is essentially a matter of number relations. The reasonableness of the estimated answer is a function of the awareness of the pupil as to what size number would satisfy the relationships specified in the problem. A sensing of the probable answer to a problem is the outcome of a good deal of experience in relating numbers through the various processes possible under our number system. Given an understanding of what is required in the problem, the judgment as to a reasonable answer is very much a matter of familiarity with number relations.

In the second place, in carrying on the mathematical processes involved in solving a problem, a facility in handling number relations is again prominent. Errors in placing decimal points are frequently due to a lack of sensitivity to the appropriate size of a number, which could not be possible if the relationships under a number system were understood more clearly. When pupils are bothered by the seeming incongruity of the answer being smaller when a number is multiplied by a fraction, or by the answer being larger when the number is divided by a fraction, it is again an illustration of a lack of clearness in regard to number relations as they are affected by arithmetical operations. The necessary teaching in such cases is not an application of more practice and drill to produce a mechanical type of efficiency but rather a kind of teaching which results in an understanding of number relations so that a child will be clear in his thinking about whole numbers and parts of numbers.

In the third place, if the number relations necessary in problem solving are to be fully understood, pupils will have to be aware of not only the nature of our own decimal number system but also of other possible number systems. Only by an understanding of the fact that number does involve a system can the child eventually realize that any particular number operation is conditioned by the general pattern of possible number operations. If one senses the general pattern, the individual operations will take on more meaning. One needs an understanding of number relations in order to handle them effectively. This understanding should give a general control of answers to problems which, in turn, will be refined by the actual process

work to find the exact answer. In order to realize such an understanding of the number system and its attendant number relations, the schools will probably have to give more emphasis than has been customary to the mathematical aspects of arithmetic as contrasted with the purely social aspects. I would hazard a guess that the social understanding of a problem will eventually be found to be related very much to a child's general level of mental ability plus his ability to read, whereas the securing of the exact answer to the problem will be found to be a function of a child's strictly number ability. In order to secure this high level of number ability, I believe we shall have to give much more attention both in teaching and in textbooks to the understanding of the number system and of number relations.

Is It Meaning of Numbers and Their Operations?

W. B. Storm, Northern Illinois State Teachers College, DeKalb, Illinois

A man who thought he was a tough thinking man of the world, said, "mathematicians were soaring in the stratosphere long before the airplane was built." Of course he meant that they dealt in a kind of mathematical philosophy which was not useful in a practical world.

Without bothering to give any rebuttal to his statement, let us observe that university professors were not the only ones who were drawing away from reality. Elementary school teachers were making the subject of aritbmetic so abstract and bookish that many children did not connect it with everyday affairs. It was not real to them. This is a concrete world and most of the numbers used are concrete numbers.

The authors of the Sixteenth Year Book of the National Council pointed out very wisely that arithmetic has a mathematical phase and a social phase. Abstract drill does not necessarily teach the pupil either one. It takes reflective thinking and discussion to teach or learn the mathematical phase. It takes experiences with everyday affairs to learn the social phase.

If you give a test to the eighth grade in a good school system, as I did, and find that only 30% succeed in dealing with social situations you will be inclined toward the conclusion that we are failing to teach the social phase of arithmetic to 70% of the pupils.

How can this condition be improved? Encourage teachers,

help teachers, demand of teachers that they develop two procedures in their everyday teaching.

First—the teaching of meaning of numbers and processes when they are first introduced and repeat the teaching of meaning at later levels if necessary.

Second—use an abundance of material drawn from life outside of school. Have pupils help secure this arithmetic material.

Successful primary teachers have been using objects and have made an energetic effort to develop concepts *but* as a group they undervalue the thinking that the child must learn to do and they overvalue the writing and manipulation of symbols. Symbols are supposed to represent ideas. If the pupil has not yet acquired number ideas, why should he use number symbols?

Some teachers have an unfortunate habit of formalizing everything that they touch, including social materials. For such teachers the only hope of salvation lies in visiting teachers who know how to manage a school room so that it is like life outside of school. In such a room it is hard to tell where book materials stop and social materials begin.

In the lower grades the problem solving is where and when to use the numbers and number processes they have learned. If they know when to count and when to add or subtract in answering the question "how many" they know the social phase of their learning. How can they know which process to use unless the distinctive characteristics of each process have been recognized through explanations and illustrations?

Teachers in the third and fourth grades would do well to make the meaning of multiplication so clear that even a third grade child will know that to get the sum of numbers that are of different size, adding is the operation to use but if the numbers are alike multiplication may be used. Through many illustrations meaning is finally put into the statement that multiplication is the process of getting the sum of like numbers.

Why not work part of the time on a list of local number situations designating in which adding must be used and in which multiplication may be used?

When a class of college girls cannot tell why we start writing the second partial product in ten's place nor why we add the two partial products, they are short on number theory. When 80% of eighth graders cannot solve a problem giving the bushels of potatoes used by a family of seven during a period of six months and asking how many bushels a family of five would use, they

are short on how to use multiplication and division.

When a teacher tries to teach multiplication and division without giving attention to groups and grouping, she is not only neglecting good theory but good practice.

When a teacher does not realize that comparing numbers is almost as common as writing numbers she is not likely to teach the great social value of fractions and per cents. How easy it is to learn that $\frac{3}{4}$ expresses the relation between 3 and 4; that my rent written as the numerator and my monthly salary written as the denominator shows immediately what fraction of my income goes for rent.

We have no right to conclude that elementary school pupils cannot learn to use per cents until we have associated fractions, decimals, and per cents very closely in many illustrations or problems.

When teachers leave out meanings which are the very reason for the operation being invented in the first place, such as when addition may be replaced by multiplication, how can we expect pupils to recognize situations calling for its use?

A psychologist has said that to get transfer in learning the teacher must associate the learning as closely as possible with the field into which transfer is expected to reach. Using this principle we are advocating that the teaching of meaning in arithmetic be closely associated with the use of arithmetic and that in teaching problem solving the teacher stress the significant adaptation of processes to situations.

Is It Teaching?

Lucile B. Gates, Sawyer Avenue School, Chicago, Illinois

Before starting to write my paper I thought it advisable to find out what specialists in the study of mathematics had written on this subject. My attention was directed to a book written by Harry Grove Wheat, Professor of Education, West Virginia University on the Psychology and Teaching of Arithmetic. On page 140 I found this startling heading to a paragraph "The Purpose of Arithmetic in Not Teaching How To Solve Problems."

At first thought there was something puzzling if not shocking in such a statement—shocking in view of all that as a teacher of arithmetic I had been taught and trained to do in my teaching.

What Professor Wheat seemed to say was that the problem disappears in the solution and when you are through with the solution you have no problem. Is it possible that there is a juggling of the word problem or a substitution for the word question? He says on page 137, concerning the "Rate-Time—Distance" problem which puzzled the ancients that it has ceased to be a problem with our modern computational system and adds: "Though one may need to exercise care in making the necessary computation, he knows exactly what he must do to arrive at the answer to the question." He admits there is a question to be answered. But is there any difference between answering a question and solving a problem? If there was a question in the mind of the pupil to be solved by some computational process, was that question also not a problem in his mind to be solved?

There was something there to be dealt with and it does not make much difference whether you call it a problem or a question, and you are not going to get rid of problem solving by calling it by some other name. As a matter of fact the dictionary defines question as follows: "A subject of inquiry or debate, a matter to be decided; a point at issue; a problem."

From Professor Wheat's book I turned to another book by a specialist on "Learning the Three R's" by Gertrude Hildreth, Psychologist, Columbia University, and on page 473 I found the following statement:

"School children get comparatively little experience in solving problems. Educators have the notion firmly entrenched that skill in computation must come first; that only when computation is at one hundred per cent efficiency should any time be spent tinkering with problems. The result is almost the incapacity of many children to solve simple written problems. Problem solving which should be the focus of arithmetic teaching from kindergarten to high school is neglected to a subordinate position if not eliminated altogether."

Here lies the issue between two groups of specialists—much computation and little problem solving on the one hand, or much problem solving with little computation on the other. If the specialists are in such positive conflict what is a poor elementary teacher to do? It would seem from reading expressions from the writings of extreme advocates on both sides of this question that, there had grown up a partisan conflict between them, and an attitude of intolerance of each others position.

It is the conclusion of universal experience, that the mad pursuit of any extreme gets one no where except in the wrong place. There is always a golden mean between all extremes and that golden mean consists in taking the generally admitted values from each extreme and combining them into a democratic program which will be of most value to the majority.

There are wide differences among pupils, some can follow the computational process better than others without concrete examples to explain their meaning. Others must rely more upon concrete examples and problems for the illustration of the computational process.

There seems to be no question that something of the abstract principle must precede the concrete example, but how far to go before introducing the example into the teaching process, should be left to the discretion of the individual teacher and her knowledge of her pupils and the shortcomings peculiar to each one. It is my feeling that there should be large freedom given to us teachers to adjust our material to pupil difficulties and that no hard and fast program should be imposed upon us in our class work.

That our students are lacking in ability to handle simple mathematical processes and in ability to solve problems is not news to teachers of mathematics. Many persons believed that it was the war that pointed out this fact to us but that is not true —it was in evidence long before the war and in spite of our teaching will probably always be so; but it is encouraging to know that as newer and more modern methods of teaching are advanced the number of students who are becoming proficient in these two mathematical phases is also becoming larger.

On the computational side may I say briefly that we can look for still greater advancement and improvement, with more general use of the "meaning theory," namely, by teaching computational procedures by reason and not by rule, by meaning and not by drill. By so doing we rationalize our initial teaching of a process until there is evidence that the pupil has complete understanding of the process. Then and only then should he be permitted to adopt the shorter thought pattern.

In the social or problem solving phase there are four factors to consider:

1. What are the chief causes of problem failure?
2. What does correct solution call for?
3. What kinds of problems can we teach?
4. What errors can we expect?

I. Most teachers of arithmetic, I believe, will agree that the following appear to be the chief causes of failure in problem solving:

1. Failure to read the problem correctly
2. Lack of knowledge of arithmetical vocabulary
3. Using the wrong computational process
4. Guessing in computation
5. Inability to make correct judgments concerning the problem
6. Lack of knowledge of place value
7. Failure to estimate answer
8. Problems of little or no practical value
9. Lack of number knowledge such as the number of feet in a mile, quarts in a gallon and so forth.

II. What does correct solution require?

1. Reading with understanding, which means that the pupil must be able to read the problem so that he will know what he is given and what he is required to find.
2. Deciding what processes he will have to use and using them correctly
3. Estimating his answer
4. Carrying out all steps until final completion
5. Knowing when to stop, when he has arrived at the point, when he has answered the question, "To Find"
6. Proving the result.

III. What kind of problems shall we teach?

A child will learn best and remember longest that in which he sees sense; therefore we should select topics and problems that have a relation to his every day living. The 16th Year Book of the National Council of Teachers of Mathematics pp. 152 and 153 has a splendid list of 54 topics related to consumer education.

We are too apt to misuse our text books in our teaching. A good arithmetic text book is a primary source of material but its real function is to offer kinds of problems and present an outline of computational processes, each to be used in its proper place, but never to be used as the main body of problem material.

Time does not permit me to elaborate on the type of problems that may be drawn from experiences common to both teacher and pupil but in addition to the 16th Year Book previously mentioned I can refer you to three other outstanding examples.

One by Harry B. Heflin, Professor of Education, Appalachian State Teachers College, Boone, N. C. called "Arithmetic for Every Day" in the *Instructor* April, 1945; another by Leo J. Bruechner, Professor of Elementary Education, University of Minnesota, Minneapolis, Minn. entitled "Social Arithmetic" in the *Instructor* Sept. 1944; and a third also in the *Instructor* June, 1944 is an article by Foster E. Grossnickle, Professor of Mathematics, New Jersey State Teachers College, Jersey City, N. J., "Two Kinds of Problems."

Incidentally Professor Grossnickle in this same article says: "Children who are weak in problem solving, and who have almost no understanding of the process to use, frequently begin by trying to manipulate the numbers included in the problem." "The best way," he says, "to overcome this tendency is to give some problems without numbers. Then pupils must select certain key words or phrases which suggest the process to use."

After a unit of study has been decided upon by teacher, and pupils' materials and facts assembled, from them problems will be made which will be real, vital, interesting and meaningful and there will be less chance of failure in their solutions.

Fourth and last we must constantly be aware of the many wrong things our pupils can do, and the many different ways they can do them, and always be on the alert to guide them in the right direction.

OUR MOST IMPORTANT INDUSTRY

We must remember the outstanding performance of education in this war would never have been possible if it, like industry, had not had its long peacetime background and experience to call upon in the emergency. At this time, let us give a great vote of thanks to those men and women in the field of education for their splendid job. And in the years to come we should give our wholehearted support to this—our most important industry—education!—C. F. Kettering, vice president, General Motors, in a radio address.

HEART RECORD MADE WITHOUT PHOTOGRAPHY BY PORTABLE INSTRUMENT

A portable electrocardiograph that inks its record of heart action directly on paper without photographic darkroom procedures was demonstrated here.

The inventor, Paul Traugott, president of Electro-Physical Laboratories, Inc., explained that his cardiotron can be used in the home by the physician if necessary. After two years of testing, the instrument is now being placed in quantity production.

AN EXPERIMENT ON AUTOMOBILE STOPPING DISTANCES

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The Edison Institute High School, Dearborn, Michigan
and
E. E. Ensign
The Ford Motor Company

Several years ago the authors developed a series of experiments in physics to show how an automobile in motion obeys the laws of mechanics. These experiments were used at The Edison Institute High School prior to the war. They were suspended during the war, but have been resumed this fall. In the meantime the authors have devised new apparatus for one of these experiments, and it is the purpose of this article to describe that apparatus and its use.

In the experiment two distances are marked off by the apparatus while an automobile is in motion; they are then measured directly on the pavement. These are (1) the distance travelled by the automobile from the instant the necessity for a stop appears until the brakes are applied, which is called the reacting distance, and (2) the distance travelled by the automobile after the brakes are applied until it comes to a stop, which is called the braking distance. Added together they give the total stopping distance.

Four to six persons ride in the car used for the test. The driver drives the car at a moderate speed. Without warning, a student in the rear seat pulls a switch which trips a pavement marking device, and at the same time rings a bell as the signal to stop. As quickly as possible the driver applies the brakes. As the stop-light circuit of the automobile is closed, it trips a second pavement marking device. Meanwhile a student on the front seat has watched the speedometer, and taken its reading at the instant the signal sounded. When the car comes to a stop, two students measure the distance from the car to the second mark on the pavement, and from the second mark to the first mark. These are the braking distance and reacting distance, respectively. The driver of the car is thus shown in a striking way the distance an automobile moves while he reacts to an emergency situation and brings the car to a stop.

Each pavement marking apparatus consists of a spring catapult having a cup in which is placed a quantity of brightly

colored starch paste. These catapults are clamped one at each end of the rear bumper of the automobile, by means of C clamps. One catapult is released by pulling a string which passes through a rear window of the automobile. The switch which operates the stop-signal bell is attached to the end of this string, so that a pull on the handle of the switch closes the bell circuit and trips the catapult simultaneously. The construction of the switch is shown in Figure 1. The spacing of the contacts is adjusted by bending, until a moderate pull on the handle causes

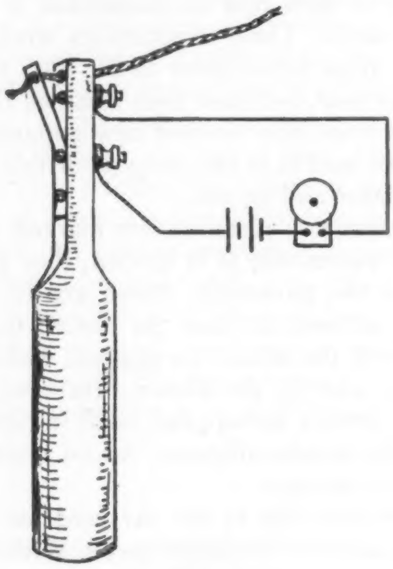


FIG. 1

the bell to ring and the catapult to trip simultaneously. The second catapult is released by an electromagnet connected to the stop-light circuit. The glass and bulb are removed from the stop-light. One wire from the magnet is fitted with a plug such as is used for attaching "trouble lamps" or similar accessories. The other wire from the magnet ends in a spring-jaw battery connector so that it may be grounded to the bumper for the return circuit. The armature of the magnet pushes a rod upward to work the release on the catapults. Figure 3 shows the magnetic release on one of the catapults. The string for manual release is shown in Figure 2.

The catapults themselves are built from a pair of large rat traps. These must be mounted in a horizontal position on the

car bumper, as shown in Figures 2 and 3. The edge of each trap is screwed to a board of suitable size, and braces placed beneath. The cross-arm which carries the paste cup is a strip of 20 gauge sheet steel bent double and bolted in place. The paste cup is a ketchup-bottle top bolted to the cross arm in a position to clear the base of the trap when it is sprung. The electromagnet is a 6-volt relay. A strip of metal is fastened to its armature and bent so as to project through a hole in the base of the

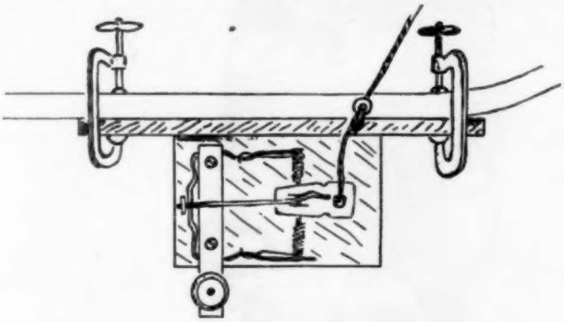


FIG. 2

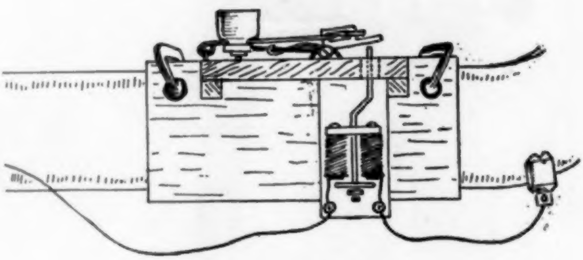


FIG. 3

trap. When a current passes through the magnet the armature rises, pushing the metal strip against the trigger of the trap.

The paste used for marking the pavement consists of approximately 45 grams of starch boiled in 1000 milliliters of water. Lead chromate gives it a bright yellow color which is easily seen.

Table I shows the results obtained by eight members of the 1945–46 physics class who acted as drivers in the experiment. In Table II speeds have been converted to feet per second and the reaction times and accelerations computed for each individual and for the group as a whole. The reaction time and acceleration are of course independent of the speed, and may

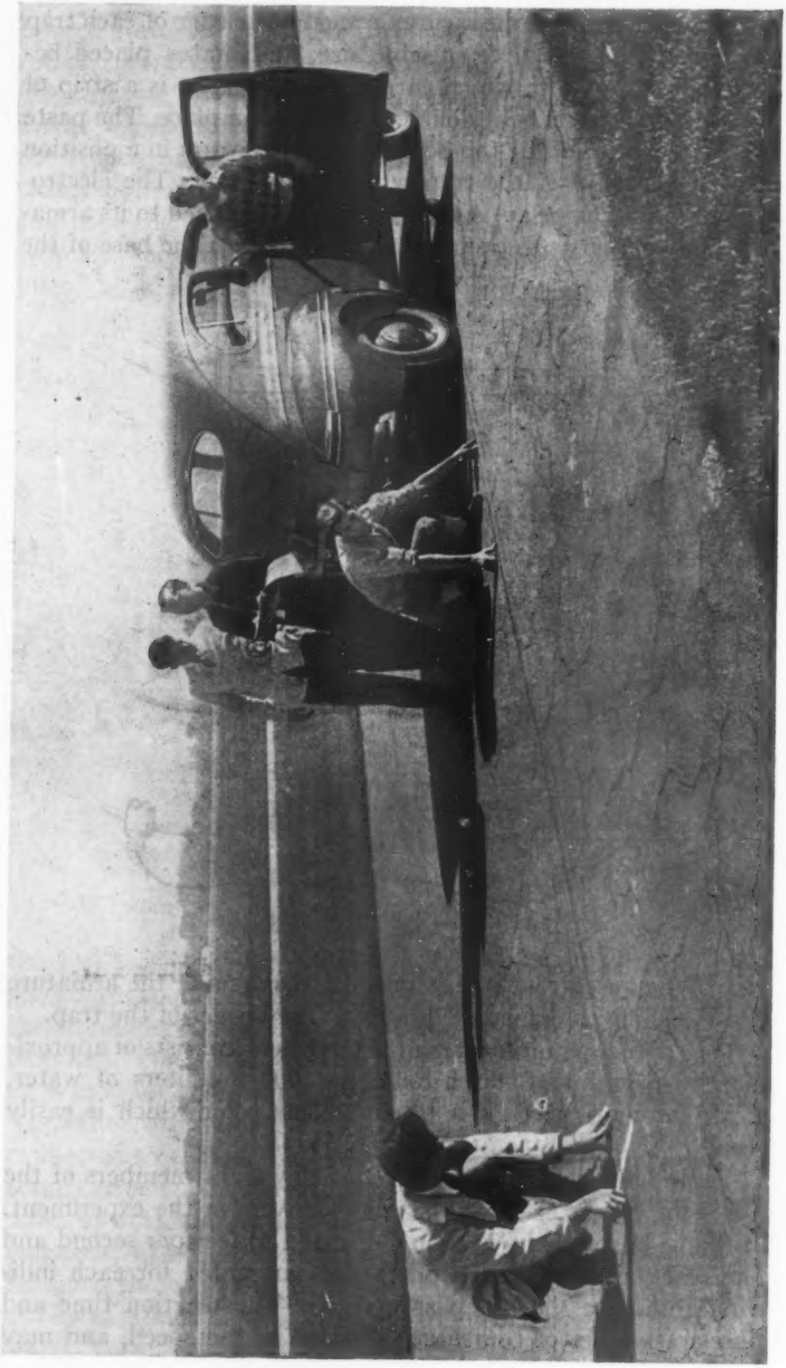


FIG. 4

be used as a basis for computing the reacting distance, braking distance and total stopping distance at any speed.

TABLE I. REACTING, BRAKING AND TOTAL STOPPING DISTANCES

Student	Speed (mi./hr.)	Reacting Distance	Braking Distance	Total Stopping Distance
A	20	18′8″	24′8″	43′4″
B	19	14′5″	14′7″	29′0″
C	23	24′2″	23′8″	47′10″
D	33	28′2″	50′0″	78′2″
E	34	24′8″	53′0″	77′8″
F	34	22′2″	57′5″	79′7″
G	28	19′9″	34′2″	53′11″
H	34	32′3″	55′2″	87′5″

TABLE II. REACTION TIMES AND ACCELERATIONS

Student	Speed (ft./sec.)	Reaction time—seconds	Acceleration (ft./sec.^{2})
A	29	.64	18
B	28	.52	27
C	34	.71	24
D	48	.58	24
E	50	.51	24
F	50	.45	22
G	41	.48	23
H	50	.65	25
Mean for 8 students		.57	23.4

Now, what are some of the reasons for teaching accelerated motion in this way? The experience of the authors indicates that the experiment succeeds in impressing certain facts about accelerated motion in such a way that they will be retained. Moreover, these facts are of immediate and long-continuing usefulness, because they are taught in their application to an important part of the student's present and future daily life. Students accept the results without question, and immediately transfer them to their out-of-school activities. Not less important is the fact that they receive a valuable lesson in safety, in a field where such lessons are badly needed. Finally, students are keenly interested in the performance of the experiment, the taking of data, and computing results, and are "sold" on physics as a study of practical value to themselves.

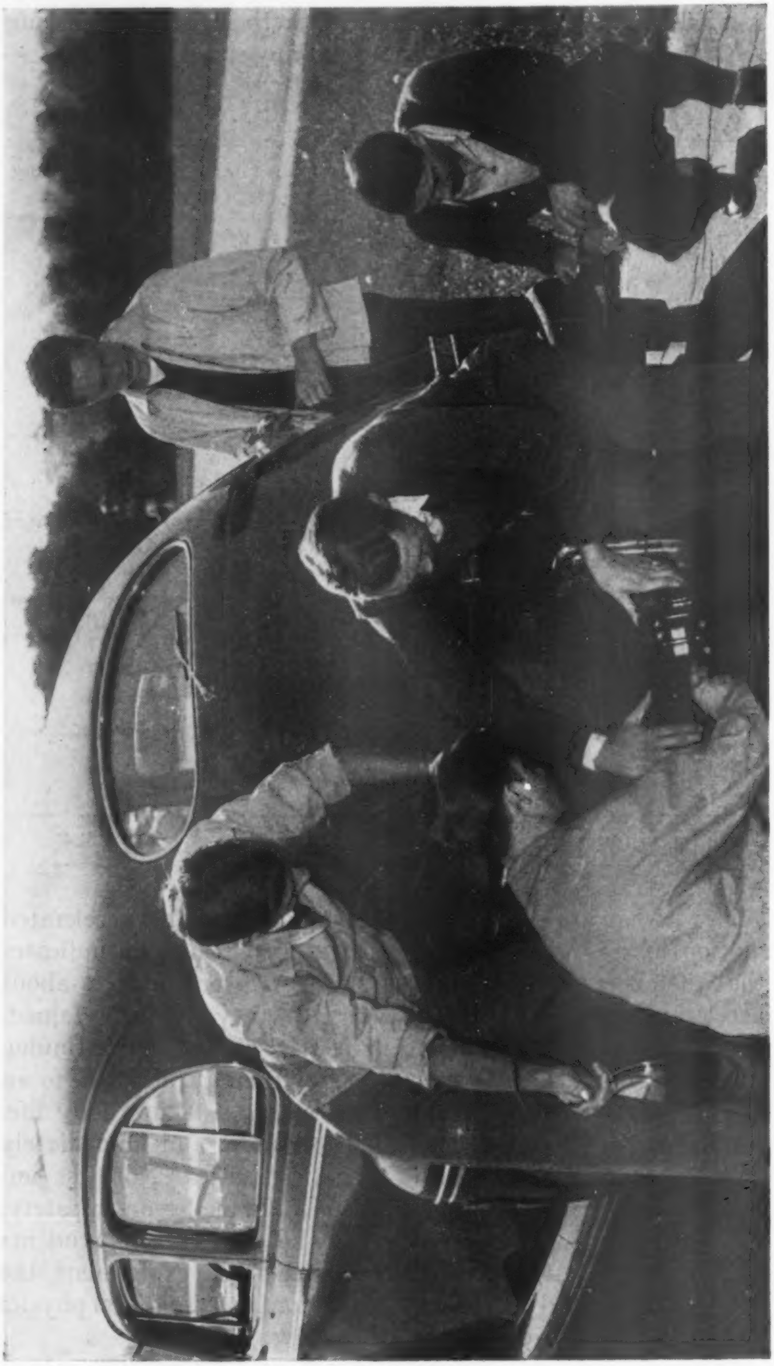


FIG. 5

ELASTICITY OF ELEMENTARY FUNCTIONS

Part 3. Application to Series

J. S. GEORGES
Chicago City Junior College: Wright Branch

The average elasticity of $f(x)$ with respect to x was defined in (1) as the ratio $x\Delta f/f\Delta x$. Interesting results can be obtained when we apply the equation to the case where $\Delta x=1$, that is, where $f(x)$ is a function of an integral variable.

Let

$$f(x)=ax(x+1)(x+2)\cdots(x+r),$$

then

$$f(x+1)=a(x+1)(x+2)(x+3)\cdots(x+r+1),$$

and

$$\Delta f=f(x+1)-f(x)=a(r+1)(x+1)(x+2)\cdots(x+r).$$

That is,

$$\Delta f=(r+1)f/x,$$

and

$$\epsilon_x(f)=x\Delta f/f=r+1,$$

since in $\Delta x=1$.

Conversely, let

$$\epsilon_x(f)=r+1,$$

then

$$\epsilon_x(f)=[x(r+1)/x](f/f).$$

Hence,

$$\Delta f=(r+1)f/x,$$

or,

$$xf(x+1)-xf(x)=(r+1)f(x).$$

Hence,

$$xf(x+1)=f(x)(x+r+1),$$

and

$$f(x)=ax(x+1)(x+2)\cdots(x+r).$$

Thus, if x is an integral variable taking on successively the values 1, 2, 3, $\cdots$, n, and if $f(x)=ax(x+1)(x+2)\cdots(x+r)$, then

$$\Delta f=a(r+1)(x+1)(x+2)\cdots(x+r);\quad \epsilon_x(f)=r+1 \tag{14}$$

and conversely.

We may apply (14) to the summation of the following classical finite series.

Example 11. Let us evaluate the series $1+2+3+\cdots+n=S$. Here $\Delta S=n+1$. Hence, $r=1$, and $\Delta S=1/2[2(n+1)]$. Hence, $a=1/2$, and $S=1/2[n(n+1)]$. That is,

$$1+2+3+\cdots+n=1/2[n(n+1)]. \tag{15}$$

The elasticity of the series (15) is 2. Formula (15) can be derived directly by means of the elasticity as follows. For $n=2$, $S=1+2=3$, and $\Delta S=3$. Hence $\epsilon_n(S)=n\Delta S/S=2$. Similarly, for $n=3$, $S=6$, $\Delta S=4$, and $\epsilon_n(S)=2$. Applying induction, we have $\epsilon_n(S_k)=2$. Hence $S_k=k(k+1)/2$, since $\Delta S_k=k+1$. Now $\epsilon_n(S_{k+1})=(k+1)(k+2)/S_{k+1}=(k+1)(k+2)/(S_k+k+1)=[(k+1)(k+2)]/[(k(k+1)/2)+k+1]=2$. The induction is complete, and $\epsilon_n(S)=2$.

Example 12. Let us evaluate the series $S=1+3+6+\cdots+n(n+1)/2$. Here $\Delta S=(n+1)(n+2)/2$. Hence, $r=2$ and $\Delta S=1/6[3(n+1)(n+2)]$. Hence, $a=1/6$, and $S=n(n+1)(n+2)/6$. That is,

$$1+3+6+\cdots+n(n+1)/2=n(n+1)(n+2)/6. \tag{16}$$

The elasticity of the series (16) is 3.

Example 13. Let us evaluate the series $S=\Sigma n(n+1)(n+2)$. Here $\Delta S=(n+1)(n+2)(n+3)$, and $r=3$. Hence $\Delta S=1/4[4(n+1)(n+2)(n+3)]$, and $a=1/4$. Thus, $S=n(n+1)(n+2)(n+3)/4$. That is,

$$1.2.3+2.3.4+\cdots n(n+1)(n+2)=n(n+1)(n+2)(n+3)/4 \tag{17}$$

The elasticity of the series (17) is 4.

Example 14. In general, let us evaluate the series $\Sigma(r+n-1)!/(n-1)!$ Here $\Delta S=(r+n)!/n!=(n+1)(n+2)\cdots(n+r)$. Hence by (14), $a=1/(r+1)$, and by the converse of (14), $S=n(n+1)(n+2)\cdots(n+r)/(r+1)$. That is

$$\Sigma(r+n-1)!/(n-1)!=(r+n)!/(r+1)(n-1)!. \tag{18}$$

The same type of analysis may be used in the evaluation of

the finite series of the form Σn^r. In this case, if $\Delta f=(x+1)^r$, then $f(x)$ is a polynomial function of the integral variable x of degree $r+1$. Now the average elasticity of $f(x)$ is $\epsilon_x(f)=x\Delta f/f$. Hence, $\epsilon_x(f)$ is a rational fractional function of the form $g(x)/h(x)$, with g and h of the same degree. From the equation $x(x+1)^r h(x) =f(x)g(x)$, it follows that if g and h are relatively prime, $f(x)$ must have the factors x and $x+1$. Since for $r=1$, $\epsilon_x(f)$ is a constant, as seen from (15), it follows that in general $f(x)$ has only the factor $x+1$ in common with Δf. That is,

$$\text{if} \quad \Delta f=(x+1)^r, \quad \text{then} \quad f(x)=x(x+1)\theta(x) \tag{19}$$

where θ is of degree $r-1$.

Example 15. Let us evaluate the series Σn^2. Since $\Delta S=(n+1)^2$, we have $S=n(n+1)(ax+b)$. That is θ is of the first degree. From $S=1$ when $n=1$, and $S=5$ when $n=2$, we have $a=1/3$ and $b=1/6$. Hence,

$$1^2+2^2+3^2+\cdots+n^2=1/6n(n+1)(2n+1). \tag{20}$$

Example 16. For the series Σn^3, $\Delta S=(n+1)^3$. Hence, $S=n(n+1)(ax^2+bx+c)$. That is, θ is of the second degree. From $S=1$ when $n=1$, $S=9$ when $n=2$, and $S=36$ when $n=3$, we have

$$1^3+2^3+3^3+\cdots+n^3=1/4n^2(n+1)^2. \tag{21}$$

Example 17. For the series Σn^4, $\Delta S=(n+1)^4$. Hence, $S=n(n+1)(ax^3+bx^2+cx+d)$. From $S=1$ when $n=1$, $S=17$ when $n=2$, $S=98$ when $n=3$, and $S=354$ when $n=4$, we have

$$1^4+2^4+3^4+\cdots+n^4=1/30n(n+1)(6n^3+9n^2+n-1). \tag{22}$$

Example 18. For the series Σn^5, we have $\Delta S=(n+1)^5$. Hence, $S=n(n+1)(ax^4+bx^3+cx^2+dx+e)$. Substituting successively $n=1, 2, 3, 4, 5$, we have

$$1^5+2^5+3^5+\cdots+n^5=1/12n^2(n+1)^2(2n^2+2n-1). \tag{23}$$

Series (20), (21), (22), and (23) are special cases of the general series Σn^r. For large values of r we may use the following general formula which applies to the case when Δf is any polynomial function of x of degree r and $f(x)$ is a polynomial function of degree $r+1$.

Let

$$\Delta f=a_1x^r+a_2x^{r-1}+\cdots+a_rx+a_{r+1},$$

and

$$f(x)=A_0x^{r+1}+A_1x^r+\cdots+A_rx+A_{r+1},$$

then

$$A_m=\frac{a_{m+1}-\sum_0^{m-1}\binom{r+1-i}{m+1-i}A_i}{r+1-m} \tag{24}$$

where the binomial coefficient represents the combination of $r+1-i$ taken $m+1-i$ at a time. Formula (24) does not define the value A_{r+1}. That value is obtained from an initial condition satisfied by $f(x)$.

The proof of (24) is as follows.

$$f(x+1)=A_0(x+1)^{r+1}+A_1(x+1)^r+\cdots+A_r(x+1)+A_{r+1}.$$

Hence

$$\begin{aligned}\Delta f&=f(x+1)-f(x)\\&=\binom{r+1}{1}A_0x^r+\left[\binom{r+1}{2}A_0+\binom{r}{1}A_1\right]x^{r-1}\\&\quad+\left[\binom{r+1}{3}A_0+\binom{r}{2}A_1+\binom{r-1}{1}A_2\right]+\cdots\\&\quad+(A_0+A_1+\cdots+A_r).\end{aligned}$$

Equating the coefficients of the like powers of x in $f(x+1)-f(x)$ and Δf, we have (24).

Example 19. Let $\Delta f=2x^3+6x^2+9x$, then $f(x)=A_0x^4+A_1x^3+A_2x^2+A_3x+A_4$. By (24), $A_0=1/4\ a_1=1/2\ A_1=1/3(a_2-6A_0)=1$, $A_2=1/2(a_3-4A_0-3A_1)=2$, $A_3=a_4-(A_0+A_1+A_2)=-7/2$. Hence, $f(x)=1/2x^4+x^3+2x^2-7/2x+A_5$. As an example of this function, let us evaluate the series $\Sigma(2n^3+3n-5)$. Here $\Delta S=2(n+1)^3+3(n+1)-5=2n^3+6n^2+9n$. Hence, $S=1/2n^4+n^3+2n^2-7/2n+C$. Since $S=0$ when $n=1$, $C=0$, and we have

$$0+17+58+135+\cdots+(2n^3+3n-5)$$
$$=1/2(n^4+2n^3+4n^2-7n). \tag{25}$$

When you change address be sure to notify Business Manager Ray C. Soliday, P. O. Box 408, Oak Park, Ill.

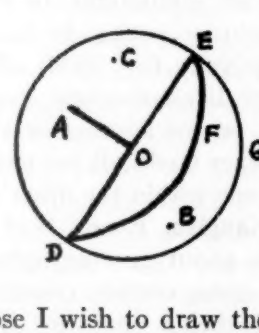
NOTES FROM A MATHEMATICS CLASSROOM

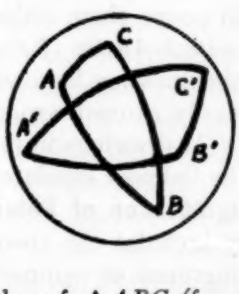
Joseph A. Nyberg
Hyde Park High School, Chicago

112. Drawings of Polar Triangles. In plane geometry a pupil is taught not to draw an equilateral or an isosceles or a right triangle when a statement deals with *any* triangle. Proofs based on such special figures may be correct but the figure is misleading. Considering the general acceptance of this principle it is surprising how many misleading drawings of polar triangles are found in textbooks; in most of them the triangles appear to be equilateral. I have even read reports which state that figures showing one triangle within its polar are acceptable, implying that the making of any other kind of figure is a difficult task not to be expected of even normal pupils. Actually, drawing the polar of a triangle is about as difficult as drawing the outline of an egg.

The text by *Strader* and *Rhoads* (John C. Winston Co., 1929) faces the issue better than most texts by stating "A given spherical triangle and its polar triangle are usually shown as one triangle entirely within the other. The pupil should understand, however, that it is not only possible to have the two triangles overlap, but also that such triangles are quite common."

If a class has a spherical blackboard it can construct some triangles on it and then copy them in the same manner that an art class draws figures of flowers or fruit. But a fairly good job can be done without a spherical blackboard.





Suppose I wish to draw the polar of $\triangle ABC$ (for simplicity in the first figure, only the vertices of the triangle are shown). To obtain the great circle arc of which A is the pole, draw AO, and draw the diameter DE perpendicular to AO. Then draw an oval through D and E, noting that the shorter AO is the

narrower will be the crescent between the oval *DFE* and the semicircle *DGE*. Next erase *AO* and *DE*, and repeat the process for points *B* and *C*. The process is so simple that there is no reason why pupils should be excused from drawing a neat and correct figure.

The correctness of the result can be tested in several ways. First, if $A'B'C'$ is the polar of ABC, then ABC is the polar of $A'B'C'$, and we examine the figure to see if the above process would lead to ABC when starting with $A'B'C'$.

A better check can be had by using the theorem: In two polar triangles an angle of either triangle is the supplement of the opposite side of the other triangle. Hence,

$$A+a'=180°$$

and

$$B+b'=180°$$

so that:

$$A+a'=B+b',$$

If A is greater than B, then a' is less than b'.

If this relation is not satisfied in the drawing, then we have a poor drawing. Due to the curvature of the arcs it may at times be difficult to decide whether a' or b' is the greater. In that case we can apply the theorem: If two arcs are unequal, the opposite angles are unequal, and so forth. These checks can be combined and written briefly as:

If $a>b>c$ then $A>B>C$, and $a'<b'<c'$, and $A'<B'<C'$.

From curiosity I have examined the drawings in a number of textbooks. When the triangles are equilateral, of course no check will prove them either right or wrong. In the text by *Sykes-Comstock-Austin* (Rand McNally Co., 1933) all are correct and the drawings have variety. In another text, every drawing was easily proved wrong by one or another or the above checks. Of 21 drawings in four other books all but one drawing were of the "almost equilateral, one within the other" type.

113. Significance of Polar Triangles. Every text in Solid Geometry includes the theorems about proving spherical triangles congruent or symmetric under certain conditions, and theorems about the greater side being opposite the greater angle, and so forth. I usually call the attention of the class to the existence of these theorems and dispose of that work in two minutes by adding "We shall not spend much time on them since there are more interesting things to consider. However,

one of the theorems is significant." I refer to the theorem: If two triangles on the same sphere are mutually equiangular, they are mutually equilateral.

This theorem is worth study because it exhibits an important difference between plane and spherical triangles. In a plane, if the respective angles of two triangles are equal, then the corresponding sides are proportional. On a sphere, if the respective angles are equal, then the corresponding sides are equal.

Other differences between plane and spherical triangles can then be brought forth: in one, the sum of the angles is 180°; in the other the sum exceeds 180°. In one, an exterior angle equals the sum of the two non-adjacent interior angles; in the other, this relation is not true.

114. Relating Solid to Plane Geometry. Besides the comparisons mentioned in the preceding paragraph, there are some others which deserve attention at one place or another in the course.

A diagonal of a parallelogram divides it into two congruent triangles. Does the plane through diagonally opposite edges of a parallelepiped divide the latter into congruent prisms?

We have a formula for the area of a triangle in terms of its sides. We should have a formula for the volume of a tetrahedron in terms of its six edges.

In plane geometry we find the area of a rectangle, and then in succession find the area of a parallelogram, a triangle, and any polygon. There is a corresponding procedure in finding volumes: first the volume of a rectangular solid, and then the volume of a parallelepiped, a triangular prism, and any prism.

When the class proves that a sphere can be inscribed in any tetrahedron it should review the work of inscribing a circle in a triangle. In plane geometry, did the class prove or assume that the first two angle bisectors intersect? Likewise, how do you prove that the planes bisecting two of the dihedral angles of the tetrahedron are not parallel? And how do you prove that the third such plane is not parallel to the intersection line of the first two? In any class I am willing to assume statements that cannot be proved and statements that can be easily proved, but I do not want the pupil to think that every statement falls into one or the other of these two categories.

When the class proves that a sphere can be circumscribed about a tetrahedron, it should review the work of circumscribing a circle about a triangle. How do you prove that the first

two planes bisecting and perpendicular to edges of a tetrahedron are not parallel, and how do you prove that the third such plane is not parallel to the intersection line of the first two?

Various schemes are used to find the volume of a sphere. The theorem of Cavalieri is growing in popularity but it does not resemble any theorem for finding areas. (A good exercise for a class is: What would be the corresponding theorem for a plane?) Since we can find the area of a circle by drawing tangents, joining their intersections to the center, adding the triangles, and using limits, we should analogously find the volume of a sphere by drawing tangent planes, passing planes through their intersections and the center, and adding the volumes of the pyramids.

Symmetric spherical triangles are proved equivalent by dividing them into isosceles triangles. Can plane triangles be divided into isosceles triangles? Why do we not study symmetric plane triangles?

It is not to be expected that a pupil will notice these similarities unless attention is called to them.

115. Solid Geometry and Freehand Sketching. In plane geometry we expect a pupil to construct each figure with compasses and a ruler. In solid geometry *construct* means merely to tell how the lines, points and planes are determined, but teachers still expect a pupil to use a ruler when drawing the figures. There are good reasons for asking the pupil in solid geometry classes to put aside his ruler and draw each figure freehand.

An engineer on a construction job is sure to carry a pad of paper but seldom carries a ruler. Yet dozens of times he needs to make a sketch of the general layout of some pipes, walls, foundations, towers, embankments, abutments, projections, intersections, supports, runways, inclined planes, levers, wedges, tanks, and a host of similar objects. In the shop and office the engineer must be able quickly to make a neat freehand sketch of any object under consideration or construction. Teachers of solid geometry are neglecting an important item unless they develop this ability in the prospective engineer. Naturally at first the drawings will be crude and far from neat, but with practice and persistence the pupil can learn to draw without using a ruler. Some former pupils have even said that this was to them the most valuable part of the course.

TOWARD A BETTER SCIENCE TEACHING PROFESSION

Lee R. Yothers
Rahway High School, Rahway, New Jersey

Secondary science department heads have as one of their duties the orientation of young teachers into the science teaching profession. This responsibility offers the opportunity to study extensively the young in-service teachers. Through close contact with them, the department head may observe the strength and, also, the inadequacies of this group. Frequently, it is found that their loyalty and conscientiousness exceed their skill as teachers.

Recently, a number of young and inexperienced teachers indicated to the writer the difficulty which this group has in getting objectives, which ethically lead to both personal and professional growth, definitely before them. Specifically, the junior teachers desired from an experienced department head a pattern of standards and activities which could be applied toward the attainment of this goal.

It may be stated, at the outset, that the choice of identity through professional years lies largely with the teacher. Obviously, lassitude results in a rule-of-thumb teacher. A type ignored because they are so familiar. In contrast to this viewpoint, a teacher may elect to achieve the satisfaction which attends a life of purpose through adequate preparation and worthwhile activities. What, then constitutes, or should, the core of a good science teacher? The following are recommended for consideration.

Graduate Study

This phase of professional training has been widely emphasized. Therefore, extended comments will not be made. It would be difficult, however, to ignore or overemphasize the additional value which graduate study gives. Also, the writer believes that future trends will be in the direction of requiring higher standards of science teachers. A point of primary importance in establishing objectives which lead to advancement, both personal and professional, is to continue the stimulus for training on the graduate level. The extent to which this field is pursued should be based upon individual and professional needs. For secondary purposes one should consider a master's

degree, *in science*, as the minimum essential. Emphasis, here, has not been placed upon the degree, but rather on the additional training, thus enabling the individual to develop an enriched scientific background and outlook. This will result in increased teaching ability and likewise would be an important factor in raising the standards of the profession.

Research

The statement, "we are living in a scientific age," is now an encyclopedic one. Perhaps, it would be more descriptive to say, "we are living on the border of an intensive and extensive international scientific research era." There is, therefore, an increasing need for science educators from kindergarten through the university to become the laymen's interpreter of science fundamentals which have been acquired from research specialists. The utilization of this scientific knowledge may be heightened through an understanding and, in addition, a fair degree of skill in using and teaching research techniques. An objective of every science teacher should be to familiarize himself, in the early era of his career, with procedures of research. While advanced research may not always be feasible, certain features of this work may be accomplished by adopting an independent study of many problems of research nature.

In all probability early studies will be of no value to anyone except the teacher. However, information and *experience* secured in this manner, over a period of years, will be followed by better and more useful teaching. Definitely both the teacher and students will gain many advantages from effort put into this work. Among them will be the following:

1. A more meaningful cooperation between research specialists and science teachers.
2. A sympathetic appreciation of the vast amount of time, energy, and money necessary for conducting scientific experimentation.
3. More intelligent interpretation to students and laymen of the results obtained through scientific research.
4. Develop a systematic analysis of problems.
5. Gain an understanding of the methods employed by accomplished research workers.
6. Develop a desire to find real facts.
7. Increased ability to instill the spirit and desire for research into capable students.

Science Teacher Organizations

Professional progressiveness demands keeping informed on the latest scientific movements and advancement. One method of assuring this is by cooperative affiliation with many science organizations. A teacher should spend much time in at least one science organization by contributing to its activities. An active affiliation will result in an increased interest in this work. Lack of membership and/or interest precludes the teacher's contact with much of the latest scientific progress.

One need not join all the science organizations. Membership, however, should include local, state, and national organizations. The latter should include an organization which is devoted to advancing the teacher's particular field of specialization and, in addition, one or more which is organized to advance all phases of science.

Membership in organizations should be considered as a reciprocal enrichment. In return for the service given by the organization, the teacher should be willing to contribute as follows:

1. Financial aid through membership.
2. Cooperate by answering questionnaires.
3. Attend conferences when they are held within a possible geographical range.
4. Participate, if possible, on programs.
5. Serve on committees when the opportunity arises.

Science Journals

Science teachers should be avid readers of science literature. They have at their disposal an excellent selection of journals which are devoted exclusively to scientific spheres. The writer wonders, however, how many avail themselves of this literature. Surely these journals are not inferior in quality; nor are their costs prohibitive. Following one's university training, science magazines become one of the most valuable instruments available for linking teachers with the latest scientific advances and procedures.

Annual Personal Program

A final suggestion may be stated with emphasis. Planning in advance, the year's program of personal activities should have a definite place in organizing one's professional objectives. The essence of good teaching is based on intelligent experiences. Ex-

perience is gained through a broad program of activities. Activities, in turn, require personal initiative. Putting into action the desire to do worthwhile things is one of the germane assets which a science teacher may have. It is advisable, therefore, at the beginning of each school year to organize personal procedures which are to be accomplished during the school year. As each item is noted, a final date should be designated for its completion. Once the program is organized there should be no let down until the last item is successfully accomplished. Failure to conduct and bring to a satisfactory conclusion every activity will reduce interest and confidence, and soon cause a blank in the schedule. On the other hand, continued effort will be highlighted by many meritorious accomplishments over a period of years.

The science teacher who pursues an active science life must inevitably gain knowledge and experience which will prove beneficial to the teacher, his students, and the profession.

PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON
State Teachers College, Kirksville, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution or proposed problem sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. N. Jamison, State Teachers College, Kirksville, Missouri.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Drawings in India ink should be on a separate page from the solution.

2. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

3. In general when several solutions are correct, the one submitted in the best form will be used.

LATE SOLUTIONS

1945, 6, 9. *Harry Siller, Washington, D. C.*

1945, 6, 7. *Paul Mount-Campbell, Rosuell, N. Mex.*

1947, 8. *Isadore Gosz, West DePere, Wis.*

1946, 7. *W. T. Talbot, Jefferson City, Mo.*

1945, 6, 7, 8, 50, *M. Kirk, Media, Pa.*

1947. *Francis L. Miksa, Aurora, Ill.*

1951. *Proposed by Clarence R. Perisho, McCook, Neb.*

For what value of n will $n!$ end in exactly 10 zeros? Also show that $n!$ cannot end in exactly eleven zeros.

Solution by W. R. Talbot, Jefferson City, Missouri

If $n!$ ends in exactly 10 zeros, it must be divisible by 2^{10} and 5^{10}. Since the powers of 2 are readily accumulated, it is necessary only to count by fives, and the value of n is found to be 45. Since the next value of n that is divisible by 5 contains 5 as a factor twice, it is seen that 50! will involve 12 zeros. In the above count by five's, observe that only the group, 21 to 25, contains 5^2.

Solutions were also offered by Harry Siller, Washington, D. C.; William E. Block, Chicago; Aaron Buchman, Buffalo, N. Y.; Edgar A. Rose, Rochester, N. Y.; Wm. A. Richards, Berwyn, Ill.; Hugo Brandt, Chicago; Brother Philip, Mont-St. Louis, Montreal; A. Struyk, Paterson, N. J., and the proposer.

1952. *Proposed by Joe Nyberg, Chicago.*

Find the volume of an icosahedron whose edge is e.

Solution by Hyman Zalosh, New York City

The plan consists of finding the volumes of the equivalent pentagonal pyramids $P-ABCDE$ and $Q-FGHKL$, and then adding the volume of the prismatoid $ABCDE-FGHKL$ found between these two pyramids.

Part I. From regular pentagon $ABCDE$,

$$R=\frac{e}{2\sin 36}.$$

From right triangle POA,

$$PO=\sqrt{e^2-R^2}=\frac{e}{2\sin 36}\sqrt{4\sin^2 36-1}$$

using the fact that

$$\sin 18=\frac{\sqrt{5}-1}{4}$$

it is true that

$$\sqrt{4\sin^2 36-1}=2\sin 18.$$

$$\therefore\quad PO=\frac{e}{2\sin 36}\cdot 2\sin 18=\frac{e}{2\cos 18}.$$

$$\text{Volume } P-ABCDE=\frac{1}{3}\cdot PO\cdot\left(\frac{1}{2}\cdot 5e\cdot\frac{e}{2}\tan 54\right)$$

$$=\frac{5e^3}{24}\cdot\frac{\tan 54}{\cos 18}.$$

Again, using the fact that

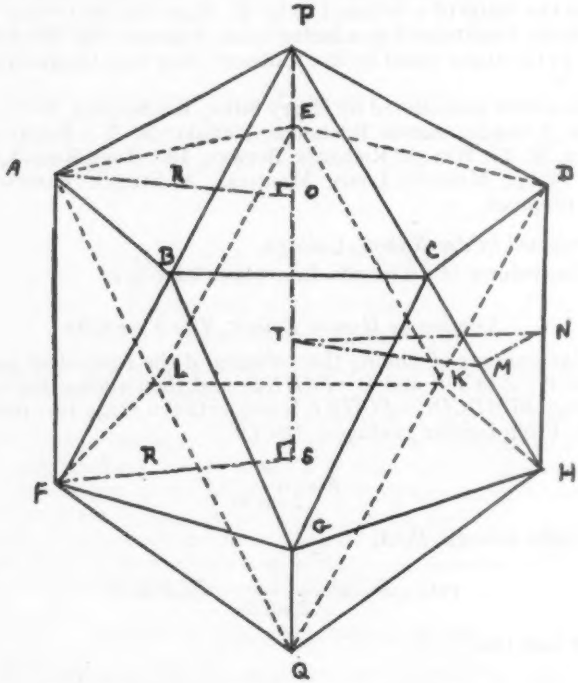
$$\sin 18 = \frac{\sqrt{5}-1}{4},$$

it can be shown that

$$\frac{\tan 54}{\cos 18} = \frac{5+\sqrt{5}}{5}$$

$$\therefore \quad \text{Volume } P-ABCDE + \text{Volume } Q-FGHKL = \frac{5e^3}{12}\cdot\left(\frac{5+\sqrt{5}}{5}\right).$$

Part II. The next few steps are aimed at finding the altitude of the prismatoid.



From triangle PBF,

$$PF = 2e \cos 36$$

From triangle PSF,

$$PS = \sqrt{4e^2 \cos^2 36 - \frac{e^2}{4 \sin^2 36}}$$

$$= \frac{e}{2 \sin 36} \cdot \sqrt{16 \sin^2 36 \cos^2 36 - 1}$$

since $\sqrt{16 \sin^2 36 \cos^2 36 - 1}$ can be shown to be $= 1 + 2 \sin 18$

$$PS=\frac{e}{2\sin 36}(1+2\sin 18)$$

$$OS=PS-PO$$

$$=\frac{e(1+2\sin 18)}{2\sin 36}-\frac{e}{2\cos 18}$$

$$OS=\frac{e}{2\sin 36}\text{ (the altitude of the prismatoid).}$$

The upper and lower bases of the prismatoid are regular pentagons. The midsection is a regular decagon whose side $(MN)=e/2$

Volume prismatoid $ABCDE-FGHKL$

$$=\frac{1}{6}\cdot OS\cdot\left[\left(\frac{5e^2}{4}\tan 54\right)+\left(\frac{5e^2}{4}\tan 54\right)+4\left(\frac{1}{2}\cdot 10\cdot\frac{e}{2}\cdot\frac{e}{4}\tan 72\right)\right]$$

$$=\frac{5e^2}{12}\cdot OS\cdot(\tan 54+\tan 72)=\frac{5e^2}{12}\cdot\frac{e}{2\sin 36}(\tan 54+\tan 72).$$

Using, for the last time,

$$\sin 18=\frac{\sqrt{5}-1}{4},$$

the above reduces to

$$\frac{5e^3}{12}\left(\frac{10+4\sqrt{5}}{5}\right)$$

$$\therefore\ \text{Total volume}=\frac{5e^3}{12}(3+\sqrt{5}).$$

Solutions were also offered by Adrian Struyk, Paterson, N. J.; Enoch D. Burton, Indianapolis, Ind.; Hugo Brandt, Chicago; Helen M. Scott, Baltimore, Md.; Margaret Joseph, Milwaukee, Wis.; Wm. A. Richards, Berwyn, Ill.; Brother Philip, Montreal, and the proposer.

1953. *Proposed by Alan Wayne, New York City.*

Editor's Note. This problem was submitted incorrectly. The c as given should have been the irrational e. Below is given the author's solution.

Find the diameter of the circle in which a chord of length e subtends an arc of length π.

Let d denote the diameter. Using radian measure, and the right triangle formed by drawing the perpendicular from the center to the chord, we have at once

$$e/d=\sin(\pi/d)$$

and setting $x=1/d$,

$$ex=\sin\pi x. \tag{1}$$

But

$$\sin\pi x=\pi x-(\pi x)^3/3!+(\pi x)^5/5!-\cdots \tag{2}$$

from the well known expansion.

Using $e=2.7183$ and $\pi=3.1416$ in the polynomial formed by substituting the three right-hand terms of (2) in (1), and discarding the trivial solution $x=0$, we obtain:

$$x^4-2.0264x^2+0.16599=0.$$

Hence $x=0.2925$, which checks in (1) to four significant figures. Therefore $d=1/0.2925=3.419$.

1954. *Proposed by M. Kirk, West Chester, Pa.*

In a right triangle ABC, show that

$$\cot\frac{A}{2}=(c+b)/a.$$

Solution by Helen M. Scott, Baltimore, Md.

$$\cot\frac{A}{2}=\frac{1+\cos A}{\sin A} \tag{1}$$

$$\sin A=\frac{a}{b} \tag{2}$$

$$\cos A=\frac{c}{b} \tag{3}$$

$$\cot\frac{A}{2}=\frac{1+\dfrac{c}{b}}{\dfrac{a}{b}}=\frac{c+b}{a}. \tag{4}$$

Solutions were also offered by Margaret L. Comstock, Ferndale, Mich.; Louis Moskowitz, Brooklyn, N. Y.; Brother Philip, Montreal; Margaret Joseph, Milwaukee, Wis.; Edith M. Warne, Columbia, Mo.; Walter R. Warne, Columbia, Mo.; Nell Munson, Cleveland, Ohio; Harry Siler, Washington, D. C.; Irmagarde Kendall, Mobile, Ala.; M. E. Hopkins, Williamstown, Mass.; Doris Craur, Cannon Falls, Minn.; Wm. A. Richards, Berwyn, Ill.; Felix John, Philadelphia, Pa.; M. I. Chernofsky, New York City; Walter R. Talbot, Jefferson City, Mo.; Hugo Brandt, Chicago; Aaron Buchman, Buffalo, N. Y.; A. W. Gordon, Clinton, Wis.; Hyman Zalosh, New York City; Edgar A. Rose, Rochester, N. Y.; and the proposer.

1955. *Proposed by Howard D. Grossman, New York City.*

If successive annual mortgage-interest payments of \$5000, \$4900, \$4800, etc., are immediately re-invested at 5% compound interest, compounded annually, find a formula for computing their total value immediately after the last payment.

Solution by A. Struyk, Paterson, N. J.

Immediately after the last payment the value is

$$v=100+200(1.05)+300(1.05)^2+\cdots+5000(1.05)^{49}.$$

$$v=100(1+2r+3r^2+\cdots+50r^{49}),\qquad r=1.05. \tag{1}$$

Let

$$s=1+2r+3r^2+4r^3+5r^4+\cdots+50r^{49}.$$

$$-2rs=-2r-4r^2-6r^3-8r^4-\cdots-98r^{49}-100r^{50}.$$

$$r^2s=r^2+2r^3+3r^4+\cdots+48r^{49}+49r^{50}+50r^{51}.$$

The sum of the preceding three equations is

$$(r^2-2r+1)s=(r-1)^2s=1-51r^{50}+50r^{51}.$$

Solving for s, and setting $r=1.05$,

$$s=\frac{1-51r^{50}+50r^{51}}{.0025},$$

so that

$$v=100s=40{,}000[1-51(1.05)^{50}+50(1.05)^{51}].$$

and this is approximately 728,044.

Miss Helen Scott, Baltimore, Md. treats (1) above as a receiving series of the form:

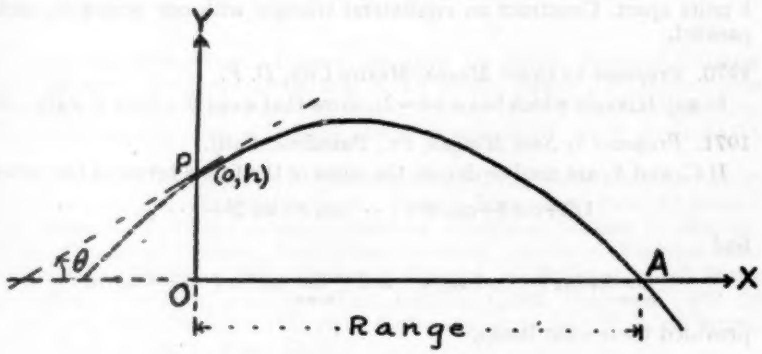
$$S=A(a+bx+cx^2+\cdots+lx^{n-1}+\cdots]$$

for which a formula found in algebra is:

$$S=A\left[\frac{a+(b-pa)x-(pl+qk)x^n-qlx^{n+1}}{1-px-qx^2}\right],$$

with S as the sum of the series, $p+q$ is the scale of relation of the coefficients. In series (1) $p=2$, $q=-1$. Also when other values are substituted, $A=100$, $a=1$, $b=2$, $k=49$ etc., the sum is found to be \$728,040.

Other solutions were offered by William A. Richards, Berwyn, Ill.; Hugo Brandt, Chicago.



Path of a Projectile

$$\text{Equation: } Y-h=X\tan\theta-\frac{gX^2}{2v^2\cos^2\theta}$$

1956. *Proposed by J. S. Miller, New Orleans, Louisiana.*

A projectile is fired from a height h above a level plane with a velocity of v at an angle of θ. Find the range.

Solution by William A. Richards, Berwyn, Illinois

The path of a projectile is a parabola, and the equation of the projectile in this problem is

$$Y-h=X\tan\theta-\frac{gX^2}{2v^2\cos^2\theta}. \tag{1}$$

To obtain the range, OA, we place $Y=0$ in (1), and solve for X. That is, we solve for X the quadratic equation

$$gX^2-(2v^2\cos^2\theta\tan\theta)X-2hv^2\cos^2\theta=0. \tag{2}$$

By using the quadratic formula, we obtain

$$X=\frac{2v^2\cos^2\theta\tan\theta\pm\sqrt{4v^4\cos^4\theta\tan^2\theta+8ghv^2\cos^2\theta}}{2g}. \tag{3}$$

Since a negative value of X would be irrelevant, we may omit the minus sign in equation (3).

Hence, using the plus sign in (3) and solving for X, we have

$$X=\text{Range}=OA=\frac{v^2\sin 2\theta+2v\cos\theta\sqrt{v^2\sin^2\theta+2gh}}{2g}.$$

Solutions were also offered by Hugo Brandt, Chicago; Edgar A. Rose, Rochester, N. Y.; Walter R. Talbot, Jefferson City, Mo.; Helen M. Scott, Baltimore, Md.; and the proposer.

PROBLEMS FOR SOLUTION

1969. *Proposed by Brother Felix John,* Philadelphia, Pa.

Given three parallels, p, q, r, with p and q, a units apart, and q and r, b units apart. Construct an equilateral triangle with one vertex on each parallel.

1970. *Proposed by Grace Marsh, Mexico City, D. F.*

In any triangle which has $a+b=2c$, show that $a\cos B-b\cos A=2(a-b)$

1971. *Proposed by Sam Morgan, Fr., Pasadena, Calif.*

If C_n and S_n are used to denote the sums of the first n terms of the series

$$1/2+\cos\theta+\cos 2\theta+\cdots,\ \sin\theta+\sin 2\theta+\cdots,$$

find

$$\lim_{n\to\infty}(c_1+c_2+\cdots+c_n)/n \quad\text{and}\quad \lim_{n\to\infty}(s_1+s_2+\cdots+s_n)/n,$$

provided there exist limits.

1972. *Proposed by Stanley Fifer.*

Any odd number of integers can always be chosen so that their sum is a perfect square.

1973. *Proposed by D. A. Wallace, St. Paul, Minn.*

Construct an isosceles trapezoid in which the altitude is a fourth proportional to the two bases and one of the equal sides.

1974. *Proposed by D. A. Wallan, St. Paul, Minn.*

Find a short way for finding the cubes of consecutive integers by addition alone.

HIGH SCHOOL HONOR ROLL

The Editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in his department. Teachers are urged to report to the Editor such solutions.

For this issue the Honor Roll appears below.

1945. Vinton Hoyle and Robert Danek, Annapolis H. S., Md.; Jack Gray, Spokane (North Central), Spokane, Wash.

1951, 4. B. J. Kirby, Upper Canada College, Toronto.

1952. J. B. Mayberry, Upper Canada College, Toronto.

BOOKS AND PAMPHLETS RECEIVED

ELECTRONICS FOR ENGINEERS, Edited by John Markus and Vin Zeluff, *Associate Editors, Electronics.* First Edition. Cloth. Pages x+390. 21.5×28 cm. 1945. McGraw-Hill Book Company, Inc., 330 W. 42nd Street, New York 18, N. Y. Price $6.00

GAINING SKILL IN ARITHMETIC, by Benjamin Braverman, *Chairman of the Department of Mathematics, Seward Park High School, New York City.* Cloth. Pages viii+134. 13.5×20 cm. 1945. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $1.40.

ESSENTIAL VOCATIONAL MATHEMATICS, by Claude H. Ewing, *Supervisor of Curriculum Washburne Trade School, Chicago, Illinois,* and Walter W. Hart, *Author of Mathematics Texts.* Cloth. Pages v+266. 14.5×22 cm. 1945. C. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $1.60.

URANIUM AND ATOMIC POWER, by Jack De Ment, *Research Chemist, The Mineralogist Laboratories,* and H. C. Drake, *Editor, The Mineralogist Magazine.* Cloth. Pages x+343. 13×21.5 cm. 1945. The Chemical Publishing Company, Inc., 234 King Street, Brooklyn 31, N. Y. Price $4.00.

INDUSTRIAL ALGEBRA AND TRIGONOMETRY WITH GEOMETRICAL APPLICATIONS, by John H. Wolfe, Sc.D., *Former Director of Ford Apprentice Training, Ford Motor Company, Dearborn, Michigan;* William F. Mueller, A.B., *Director of Training, Ford Motor Company, Dearborn, Michigan;* and Seibert D. Mullikin, B.S., *Assistant Director of Training, Ford Motor Company, Dearborn, Michigan.* First Edition. Cloth. Pages xiii+389. 13×21 cm. 1945. McGraw-Hill Book Company, Inc., 330 W. 42nd Street, New York 18, N. Y. Price $2.20.

LOOKING AHEAD IN EDUCATION, Planned and Edited by J. Wayne Wrightstone, *Board of Education of the City of New York,* and Morris Meister, *High School of Science, Bronx, New York City.* Cloth. Pages xvi+151. 14.5×22 cm. 1945. Ginn and Company, Statler Building, Boston, Mass. Price $1.50.

SCIENCE AND SCIENTISTS IN THE NETHERLANDS INDIES, Edited by Pieter Honig, Ph.D., and Frans Verdoorn, Ph.D. Cloth. Pages xxii+491. 17×26 cm. 1945. Board for the Netherlands Indies, Surinam and Curacao, 10 Rockefeller Plaza, New York, N. Y. Price $4.00.

THE LAW OF CYCLE PROGRESSION AND THE SOLUTION OF THE TRISECTION PROBLEM, RULER AND COMPASS ONLY, with Euclidean Proof, by Edward Vennigerholz. Edition 1. Paper. 32 pages. 15.5×23.5 cm. 1945. Pure Science Research Associates, Box 310, Moscow, Idaho. Price $1.00.

WHAT EVERY TEACHER SHOULD KNOW ABOUT THE PHYSICAL CONDITION

OF HER PUPILS, by James Frederick Rogers, M.D., *Formerly Consultant in Hygiene, U. S. Office of Education.* Pamphlet No. 68. 19 pages. 15×23.5 cm. 1945. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price 10 cents.

BOOK REVIEWS

YOUR CAREER IN ENGINEERING, by Norman V. Carlisle, *Vocational Editor, Scholastic, the National High School Weekly.* Cloth. 253 pages. 13.5×20.5 cm. 1942. E. P. Dutton and Company, Inc., 300 Fourth Avenue, New York, N. Y. Price $2.50.

Here is a book for the high school boy who is trying to plan his future but without adequate help. He may have read a wonderful story of a great engineer who has been highly successful in building a great bridge, developing a valuable mine, or producing a new plastic. He plans to enter an engineering field, but later, after spending considerable time and money, finds that he does not have sufficient interest in the type of work required or the ability to make the necessary preparation. The author gives the important qualifications for success and the thorough preparation demanded; describes briefly the seven principal divisions of engineering; then follows with a short chapter on each one of many of the most important specialties. In each chapter he describes the work of the main subdivisions. the type of preparation required, the possibilities for employment and advancement, the nature of the various branches of work, and the salary range. Each chapter is followed by a short bibliography. The selections here are excellent, and direct the student to additional reading if he is interested.

In the second section a number of important specialties in civil, mechanical, electrical, and other branches of engineering are given special treatment. Here are descriptions of highway, sanitary, agricultural, aeronautical heating, communication, petroleum, etc. All are briefly discussed but enough is given to show the types of work, the preparation required, and the possibilities of success. Throughout the book are many photographs of engineers at work and their products. It is an excellent first book for any boy who is thinking of an engineering career.

G. W. W.

A GENERAL ACCOUNT OF THE DEVELOPMENT OF METHODS OF USING ATOMIC ENERGY FOR MILITARY PURPOSES UNDER THE AUSPICES OF THE UNITED STATES GOVERNMENT, 1940–1945, by H. D. Smyth, *Chairman, Department of Physics, Princeton University, and Consultant to Manhattan District, U. S. Corps of Engineers.* Paper. Page 182. 15×23 cm. 1945 Supt. of Documents, Washington 25, D. C. 35¢.

This report presents the administrative history of the Atomic Bomb project, and describes in rather complete detail the basic scientific knowledge on which the several developments were based. Essentially, the report is the story of the work of many thousands of scientists, engineers, workmen, and administrators—both civilian and military—whose prolonged labor, silent perseverance, and whole-hearted cooperation made possible the unprecedented technical accomplishments.

The report is composed of thirteen chapters, which deal in chronological and logical order with the initial phases of the planning and organization, and with progress made on certain definite problems during stated periods. Rather complete summaries of knowledge of nuclear physics up to June of

1940, with descriptions of the contributions of many outstanding scientists, are included. The progress of scientific knowledge at specific stages is summarized. The problems requiring solution leading toward a controllable chain reaction of military significance are described.

Chapters dealing with the organization of military and civilian personnel may be of interest to teachers only as illustrating the administrative problems involved in scientific undertakings of such magnitude.

Three chapters are devoted to the fundamental principles of and the methods used in separating isotopes. One of the last chapters describes some of the more specific problems related to construction of the actual atomic bomb. A final short chapter presents a glimpse of the thinking about future results of the project, which must have loomed as important in the minds of many of the scientists concerned.

Appendices provide considerable information regarding various scientific problems and methods involved in construction of the self-sustaining chain-reaction pile, and on methods of observing and studying fast particles.

The better-informed teachers of the physical sciences will find little new scientific information in this volume. However, the report presents a compact description of the tremendous possibilities of scientific cooperation. *Time* for December 17, 1945, states that this is one of two topical books of 1945 which measure up the year's massive events. It can enable every teacher of the sciences to become fairly conversant with the names and contributions of the many scientists mentioned.

A new and enlarged edition, including statements by the British and Canadian Governments, with numerous illustrations, under the title, Atomic Energy for Military Purposes, is published by the Princeton University Press, Princeton, New Jersey, and is highly recommended for teachers.

RAY C. SOLIDAY

BEHAVIOR CHANGES RESULTING FROM A STUDY OF COMMUNICABLE DISEASES, by John Urban, Ph.D. Cloth. Pages viii+110. 15.5×23.5 cm. 1943. Bureau of Publications, Teachers College, Columbia University, New York, N. Y. Price $1.85.

The book presents the results obtained from a classroom study in the Millburn, New Jersey Township Public High School (grades 7 to 12). The author points out that measurement in terms of gain in information is too limited in scope to be of great value in determining effective results of teaching. There is little relationship between acquisition of facts and desirable changes in behavior. He further points out that teaching ought to be directed toward very specific ends if it is to be considered successful. The specific aim of the investigation, therefore, was to test the change in behavior of the pupils with respect to facts presented in a teaching unit. The unit selected dealt with 15 communicable diseases (the most prevalent and common types). Arrangement was made for two groups of students which were similar in intelligence, general training and previous science instruction. The "experimental" group received specific instruction in the selected health unit, the "control" group studied English. An initial test on facts and behavior was given at the start, similar tests were administered at six and twelve week interims. Behaviors involved in the study were recorded in class by an unknown observer and information on these was supplemented by interviews with the individual student. Results showed that the "experimental" group made much larger gains and showed much greater retention of facts than the "control" group. The former, likewise, demonstrated effective change in behavior due to classroom experiences.

Techniques are well discussed and two complete information tests are included. Principals and teachers will find the book stimulating reading.

J. E. P.

Dice of Destiny, by David C. Rife, Ph.D. Cloth. Pages 163. 14×20 cm. 1945. Long's College Book Company, Columbus 1, Ohio. Price $1.75.

The book presents in non-technical language the phenomena found in human genetics; use of homely similes at times explain effectively complex technical problems. The author discusses such phenomena as blood types, taste, physical features, sex, health, mental traits and capacities, race, etc. He points out the relationship of genetics to democracy, saying, "The greater the heterogeneity, the greater will be the problem of democracy." He warns against race prejudice, expressing the belief that any citizen should be judged only on personal merit. Presented well is, also, the relationship between heredity and environment, especially with respect to learned abilities. In his opinion, heredity determines the potentiality and environment will determine the degree of expression which such inherited ability will find. He adds the stimulating conclusion, "None of us have reached the full limit of our capacities." The book is enlightening, entertaining, and stimulating. It should prove valuable reading to teachers, professional men dealing with human affairs, as well as to the educated persons in general.

J. E. P.

Men of Mathematics, by E. T. Bell, *California Institute of Technology.* Cloth. Pages xxii+593. Reprint Edition, 1937. Dover Publications, New York. Price $2.75.

Mr. Bell has provided within the covers of a single book a storehouse of information on the history of mathematics and the biographies of the men who pioneered the field. He has written in a non-technical, easy, lively style that carries the reader along.

The book contains accounts of the life and works of thirty-four individuals, selected because of the importance of their work for modern mathematics and the human appeal of the men's life and character. The author's purpose is to give the reader some idea of the sort of persons the men were who created modern mathematics and to lead up to some of the dominating ideas that govern vast tracts of mathematics as it exists today.

While the chapters are arranged in chronological order, the book need not be read straight through or in that order. Aside from the interests of the general reader, probably one of the most important uses of the book is in providing the teacher and student a source of excellent historical material on the life and work of a given individual and on the major ideas of interest in a given period of history. The book should be in every high school and college library and available to all teachers of mathematics. As indicated above, this edition is reprinted from the $5.00 edition published in 1937. However, the quality of the paper, printing, and binding is very good, entirely satisfactory to the average reader.

G. E. Hawkins

Principles of Radio for Operators, by Ralph Atherton, M.S. *Assistant Professor of Physics, Miami University.* Cloth. Pages x+344. 13×20.5 cm. 1945. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $3.75.

Although the cessation of hostilities has undoubtedly lowered the demand for this type of book for special training courses, it should still be of

interest to those who, lacking any background in electricity, desire to learn the principles of radio receivers and transmitters and their component parts. The book is well adapted for self-study or for use in elementary radio courses in either high school or college. Although the text is an outgrowth of the author's experience in training radio operators for the Navy, it need not necessarily be confined to the use of operators, but is well designed for the comprehension of anyone desiring to learn the fundamentals of electricity and the function of parts and circuits used in modern radios.

The book makes no assumptions of previous knowledge in the electrical field but teaches the fundamental electrical principles needed for the understanding of radio, and applies these principles to the functioning of each part of the radio. The principles are well illustrated with schematic diagrams and the special parts in which the principle is applied are shown in picture form so that the student may recognize them in the actual radio circuit. The work is divided into sixteen chapters or lessons which aids its adaptability to a sixteen weeks or a one semester course. Each chapter contains several suggested demonstrations which may well be used as laboratory experiments where the simple apparatus is available. Each lesson closes with a review test. An excellent list of teaching helps such as films and slides is given for each chapter. An appendix of common symbols, color codes, vacuum tubes and socket connections add to the practical value of the book.

It is hoped that texts of this type, that tell the story of radio in so elementary a manner, may help students find that radio theory not only loses all its mystery and difficulty but also becomes understandable and fascinating, so that elementary radio courses may find their rightful place in the postwar curriculum.

H. R. Voorhees
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Elementary Mathematics from an Advanced Standpoint, Arithmetic-Algebra-Analysis, by Felix Klein. Translated from the third German edition by E. R. Hedrick, *Professor of Mathematics in the University of California at Los Angeles* and C. A. Noble, *Professor of Mathematics in the University of California at Berkeley.* Cloth. Pages ix+274. 14×22 cm. 1945. Dover Publications, New York. Price $3.50.

Although the English translation of this classic was first published some dozen years ago, the book is not as widely known as its worth might merit. It is doubtful if any person was as well fitted as Klein to explain the more advanced material in a subject field in such a manner as to make it of value to teachers on the secondary and junior college level. Throughout the book stress is laid on the effect this somewhat more advanced material should have on instruction.

Among the many items which should be of value might be mentioned work on number theory (much of which can be followed by a bright high school student); the theory and construction of calculating machines; an excellent discussion of the rule of signs in algebra (with emphasis that this rule cannot be proved); some excellent historical material on various phases; proofs of the transcendence of the numbers e and π; trigonometric series.

There are many references to source material, almost exclusively to material in German. The book itself covers material through the calculus, with some reference to material not usually found in high school or college courses.

If this book is not already in your school library, it should be placed there—if possible it should be added to your personal library.

CECIL B. READ
University of Wichita

THE COMET OF 1577: ITS PLACE IN THE HISTORY OF ASTRONOMY, by C. Doris Hellman, Ph.D. Cloth. Pages 317+171. 14.5×22 cm. 1944. Columbia University Press, New York. Price $6.00.

This book which is the result of nearly fourteen years of study and research aims to show the effect of the comet of 1577 on astronomical thought. The author first discusses cometary theory to the end of the fourteenth century emphasizing the Aristotelian views. Comets were definitely considered to be atmospheric phenomena. The importance of comets lay in their use in weather prognostication and in their portent as omens.

Continued observation and attempts at interpretation kept interest alive and during the period, 1400–1577, a major contribution to cometary theory was the discovery that the tail is always opposed to the sun. The new star of 1572, studied intensively by men such as Tycho Brahe, gave added impetus to astronomical observation. Since the nova showed no parallax, Tycho believed it to be in the region of the fixed stars. With the background of such observations, Dr. Hellman feels that the achievements of the observers of the comet of 1577 rose to new levels.

The parallax of this comet received special attention. As seen by Hagecius at Prague and by Tycho at Hveen, the comet showed a difference of only one or two minutes of arc proving to Tycho that it was beyond the lunar orbit. The many quoted descriptions of the comet stress the dates of appearance and disappearance, the path through the heavens, the changing brightness and the changing direction of its tail.

In the discussion of observers who found a large parallax, much space is devoted to the work of Hagecius whose descriptions were unusually good but who claimed to detect about five degrees parallax, placing the comet closer to the earth than the moon's orbit.

Several chapters review the writings of those qualified as observers but not attempting to measure the comet's distance, the treatises of preachers, poets and astrologers, and general tracts.

In a well written conclusion, the author states that cometary observation was put on a new and sounder footing. Careful observations in large numbers and from widely separated places gave a relatively small parallax and comets could no longer be considered to be atmospheric phenomena. The fear of comets prevailed as did the theological implications which were the subject of many clerics. The written works on this comet gave "a complete picture of astronomers and astronomy in the last quarter of the sixteenth century." They tended to bring acceptance of the Copernican theory. The astronomers showed their ability to observe, to make conclusions and to advance in understanding beyond the time-honored accepted beliefs.

Extensive foot-notes take up many pages of the book proper. Much of this material relative to various writers is of biographical interest while many references are made to the detailed bibliography of tracts and treatises which makes up the last third of the printed pages.

ROBERT L. PRICE
Joliet Junior College

"There is no fate in life save such as a strong hand carves, or a weak hand mars."